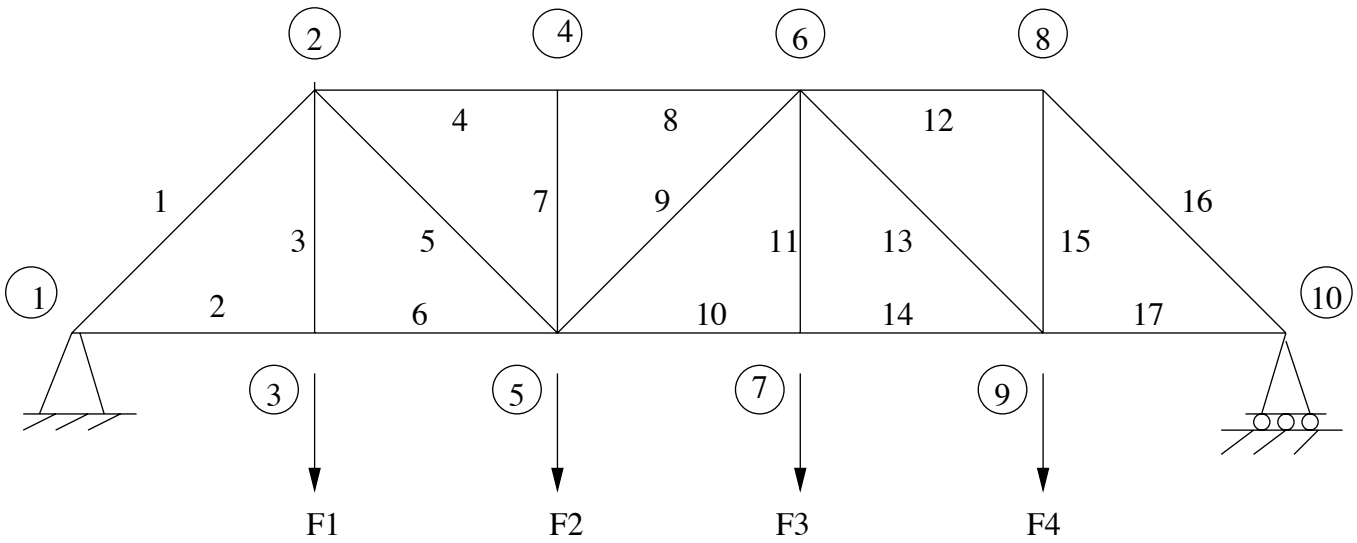
Solution of linear systems of equations: Example



KP 2 $\begin{cases} -\alpha f_1 + f_4 + \alpha f_5 = 0 \\ -\alpha f_1 + f_3 + \alpha f_5 = 0 \end{cases}$

KP 3 $\begin{cases} -f_2 + f_6 = 0 \\ -f_3 + F_1 = 0 \end{cases}$

KP 4 $\begin{cases} -f_4 + f_8 = 0 \\ f_7 = 0 \end{cases}$

KP 5 $\begin{cases} -\alpha f_5 - f_6 + \alpha f_9 + f_{10} = 0 \\ -\alpha f_5 - f_7 + \alpha f_9 + F_2 = 0 \end{cases}$

KP 6 $\begin{cases} -f_8 - \alpha f_9 + f_{12} + \alpha f_{13} = 0 \\ -\alpha f_9 + f_{11} + \alpha f_{13} = 0 \end{cases}$

KP 7 $\begin{cases} -f_{10} + f_{14} = 0 \\ -f_{11} + F_3 = 0 \end{cases}$

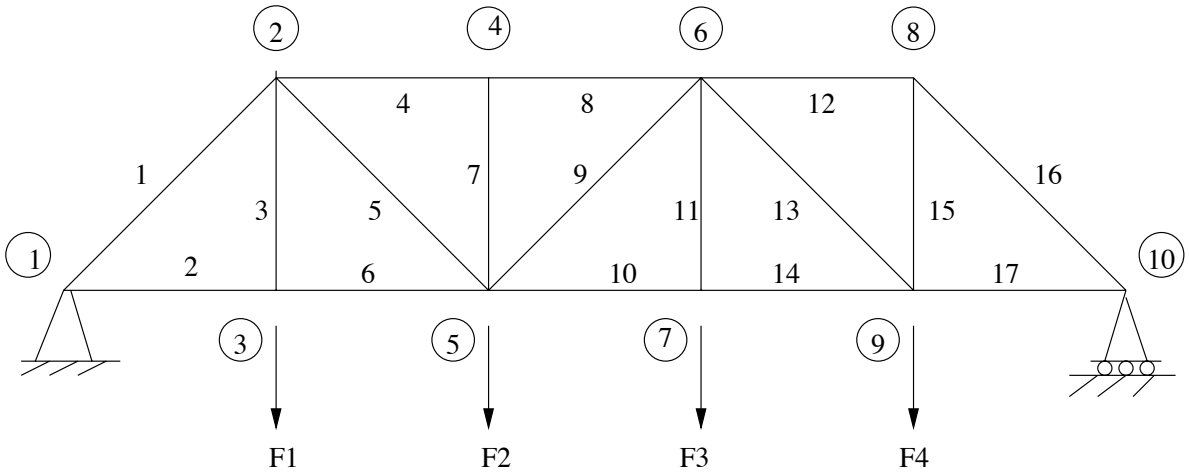
KP 8 $\begin{cases} -f_{12} + \alpha f_{16} = 0 \\ f_{15} - \alpha f_{16} = 0 \end{cases}$

KP 9 $\begin{cases} -\alpha f_{13} - f_{14} + f_{17} = 0 \\ -\alpha f_{13} - f_{15} + F_4 = 0 \end{cases}$

KP 10 $\begin{cases} -\alpha f_{16} - f_{17} = 0 \end{cases}$

$\alpha = \sin \pi/4 = \cos \pi/4$

Solution of linear systems of equations: Example



$$
\begin{pmatrix}
-\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\alpha & 0 & 1 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & \alpha & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\alpha & 0 & -1 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -\alpha & 0 & 0 & 1 & \alpha & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 0 & 1 & 0 & \alpha & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & \alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha & -\alpha & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\alpha & -1
\end{pmatrix}
\begin{pmatrix}
f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \\ f_6 \\ f_7 \\ f_8 \\ f_9 \\ f_{10} \\ f_{11} \\ f_{12} \\ f_{13} \\ f_{14} \\ f_{15} \\ f_{16} \\ f_{17}
\end{pmatrix}
=
\begin{pmatrix}
0 \\ 0 \\ 0 \\ -F_1 \\ 0 \\ 0 \\ 0 \\ -F_2 \\ 0 \\ 0 \\ 0 \\ -F_3 \\ 0 \\ 0 \\ 0 \\ -F_4 \\ 0
\end{pmatrix}
$$

Naive Gauss elimination

Problem:

$$A = \left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} & b_3 \\ \vdots & & & & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} & b_n \end{array}\right]$$

Algorithm:

for $k = 1, 2, \cdots, n-1$
  for $i = k+1, \cdots, n$
    $m_{ik} = a_{ik} / a_{kk}$
    for $j = k+1, \cdots, n$
      $a_{ij} \leftarrow a_{ij} - m_{ik} a_{kj}$
    end
    $b_i \leftarrow b_i - m_{ik} b_k$
  end
end

Naive Gauss elimination

Problem:

$$A = \left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} & b_3 \\ \vdots & & & & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} & b_n \end{array}\right]$$

$$A^{(2)} = \left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ m_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ m_{31} & a_{32} & a_{33} & \cdots & a_{3n} & b_3 \\ \vdots & & & & \vdots & \vdots \\ m_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} & b_n \end{array}\right]$$

Algorithm:

for $k = 1, 2, \cdots, n-1$
  for $i = k+1, \cdots, n$
    $m_{ik} = a_{ik} / a_{kk}$
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  end
end

Naive Gauss elimination

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$$A = \left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} & b_3 \\ \vdots & & & & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} & b_n \end{array}\right]$$

$$A^{(3)} = \left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ m_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ m_{31} & m_{32} & a_{33} & \cdots & a_{3n} & b_3 \\ \vdots & & & & \vdots & \vdots \\ m_{n1} & m_{n2} & a_{n3} & \cdots & a_{nn} & b_n \end{array}\right]$$

Algorithm:

for $k = 1, 2, \cdots, n-1$
  for $i = k+1, \cdots, n$
    $m_{ik} = a_{ik} / a_{kk}$
    for $j = k+1, \cdots, n$
      $a_{ij} \leftarrow a_{ij} - m_{ik} a_{kj}$
    end
    $b_i \leftarrow b_i - m_{ik} b_k$
  end
end

Naive Gauss elimination

Problem:

$$A = \left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} & b_3 \\ \vdots & & & & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} & b_n \end{array}\right]$$

$$A^{(n)} = \left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ m_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ m_{31} & m_{32} & a_{33} & \cdots & a_{3n} & b_3 \\ \vdots & & & & \vdots & \vdots \\ m_{n1} & m_{n2} & m_{n3} & \cdots & a_{nn} & b_n \end{array}\right]$$

Algorithm:

for $k = 1, 2, \cdots, n-1$
  for $i = k+1, \cdots, n$
    $m_{ik} = a_{ik} / a_{kk}$
    for $j = k+1, \cdots, n$
      $a_{ij} \leftarrow a_{ij} - m_{ik} a_{kj}$
    end
    $b_i \leftarrow b_i - m_{ik} b_k$
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Naive Gauss elimination

Problem:

$$A = \left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} & b_3 \\ \vdots & & & & \vdots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} & b_n \end{array}\right]$$

$$A^{(n)} = \left[\begin{array}{ccccc|c} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & b_1 \\ m_{21} & a_{22} & a_{23} & \cdots & a_{2n} & b_2 \\ m_{31} & m_{32} & a_{33} & \cdots & a_{3n} & b_3 \\ \vdots & & & & \vdots & \vdots \\ m_{n1} & m_{n2} & m_{n3} & \cdots & a_{nn} & b_n \end{array}\right]$$

Algorithm:

for $k = 1, 2, \cdots, n-1$
  for $i = k+1, \cdots, n$
    $m_{ik} = a_{ik}/a_{kk}$
    for $j = k+1, \cdots, n$
      $a_{ij} \leftarrow a_{ij} - m_{ik} a_{kj}$
    end
    $b_i \leftarrow b_i - m_{ik} b_k$
  end
end

$x_n = b_n/a_{nn}$
for $k = n-1, \cdots, 1$
  $x_k = (b_k - \sum_{s=k+1}^{n} a_{ks} x_s)/a_{kk}$
end