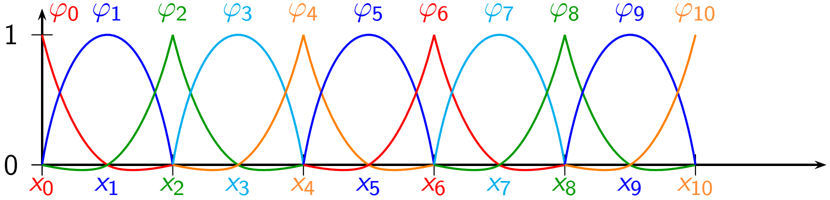
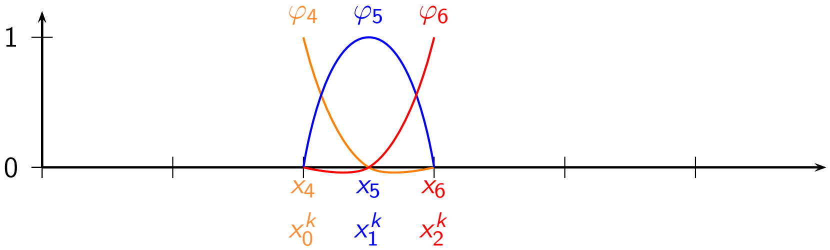
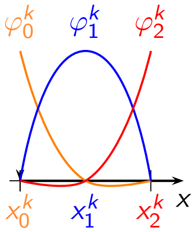

φ_0 φ_1 φ_2 φ_3 φ_4 φ_5 φ_6 φ_7 φ_8 φ_9 φ_{10}
1
0
x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10}

Pick out element $k = 3$

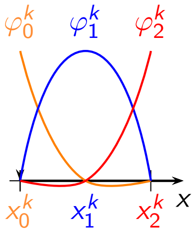


Local-to-global mapping: $\theta(k, \alpha) = 2(k-1) + \alpha$, $\alpha = 0, 1, 2$.

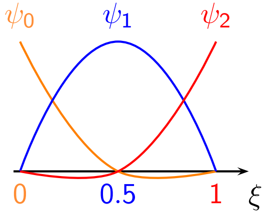
Element K_k:



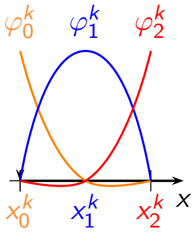
Element K_k:



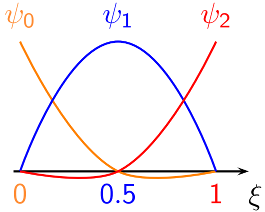
Reference element $\hat{K}$:



Element K_k:



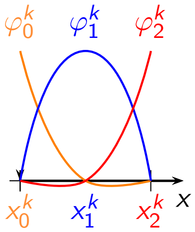
Reference element $\hat{K}$:



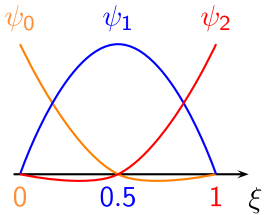
Shape functions:

$$\psi_0(\xi) = 2(\xi - \tfrac{1}{2})(\xi - 1)$$
$$\psi_1(\xi) = -4\xi(\xi - 1)$$
$$\psi_2(\xi) = 2\xi(\xi - \tfrac{1}{2})$$

Element K_k:



Reference element $\hat{K}$:



Mapping from $\hat{K} \Leftrightarrow K_k$:

$$\varphi_\alpha^k(x) = \psi_\alpha(\xi(x)) = \psi_\alpha(\Phi_k^{-1}(x)), \quad \alpha = 0, 1, 2$$

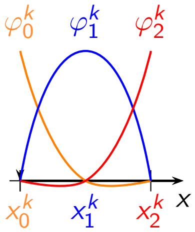
$$x(\xi) = \Phi_k(\xi) = x_0^k + h_k\xi$$

$$\xi(x) = \Phi_k^{-1}(x) = (x - x_0^k)/h_k, \quad h_k = x_2^k - x_0^k$$

Shape functions:

$$\psi_0(\xi) = 2(\xi - \tfrac{1}{2})(\xi - 1)$$
$$\psi_1(\xi) = -4\xi(\xi - 1)$$
$$\psi_2(\xi) = 2\xi(\xi - \tfrac{1}{2})$$

Element K_k:



$$\tilde{A}^k_{h,\alpha,\beta} = a(\varphi^k_\alpha, \varphi^k_\beta)|_{K_k} = \int_{x_0^k}^{x_2^k} \frac{d\varphi^k_\alpha}{dx}\frac{d\varphi^k_\beta}{dx}dx$$

$$= \int_0^1 \left(\frac{d\Psi_\alpha}{d\xi}\frac{d\Phi^{-1}}{dx}\right)\left(\frac{d\Psi_\beta}{d\xi}\frac{d\Phi^{-1}}{dx}\right)\frac{d\Phi}{dx}d\xi$$

$$= \frac{1}{h_k}\int_0^1 \frac{d\Psi_\alpha}{d\xi}\frac{d\Psi_\beta}{d\xi}d\xi$$

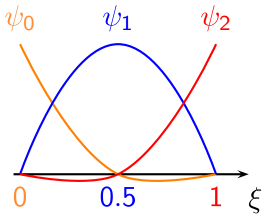
Mapping from $\hat{K} \Leftrightarrow K_k$:

$$\varphi^k_\alpha(x) = \psi_\alpha(\xi(x)) = \psi_\alpha(\Phi_k^{-1}(x)), \quad \alpha = 0, 1, 2$$

$$x(\xi) = \Phi_k(\xi) = x_0^k + h_k\xi$$

$$\xi(x) = \Phi_k^{-1}(x) = (x - x_0^k)/h_k, \quad h_k = x_2^k - x_0^k$$

Reference element $\hat{K}$:



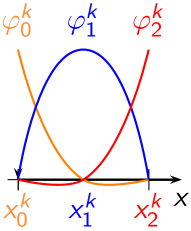
Shape functions:

$$\psi_0(\xi) = 2(\xi - \tfrac{1}{2})(\xi - 1)$$

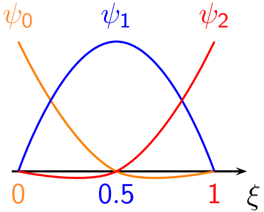
$$\psi_1(\xi) = -4\xi(\xi - 1)$$

$$\psi_2(\xi) = 2\xi(\xi - \tfrac{1}{2})$$

Element K_k:



Reference element $\hat{K}$:

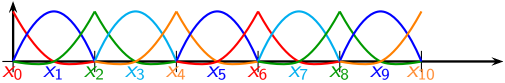


Element matrix $\tilde{A}_h^k$:

$$\tilde{A}_{h,\alpha,\beta}^k = \frac{1}{h_k} \int_0^1 \frac{d\Psi_\alpha}{d\xi} \frac{d\Psi_\beta}{d\xi} d\xi$$

$$\tilde{A}_h^k = \frac{1}{h_k} \begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix}$$

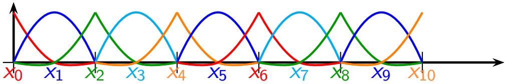
$$\psi_0(\xi) = 2(\xi - \tfrac{1}{2})(\xi - 1)$$
$$\psi_1(\xi) = -4\xi(\xi - 1)$$
$$\psi_2(\xi) = 2\xi(\xi - \tfrac{1}{2})$$



$$\tilde{A}_h^k = \frac{1}{h_k}\begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix}$$

$$i = \theta(k,\alpha) = 2(k-1)+\alpha, \qquad \alpha = 0,1,2, \qquad h_k = 2h$$

$$\tilde{A}_h = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

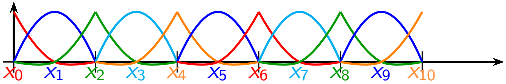


$$\tilde{A}_h^k = \frac{1}{h_k}\begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix}$$

$$i = \theta(k, \alpha) = 2(k-1) + \alpha, \qquad \alpha = 0, 1, 2, \qquad h_k = 2h$$

k=1

$$\tilde{A}_h = \frac{1}{2h}\begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

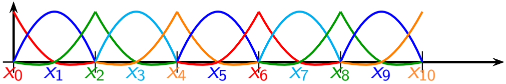


$$\tilde{A}_h^k = \frac{1}{h_k}\begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix}$$

$$i = \theta(k,\alpha) = 2(k-1) + \alpha, \qquad \alpha = 0,1,2, \qquad h_k = 2h$$

k=2

$$\tilde{A}_h = \frac{1}{2h}\begin{pmatrix}
\frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

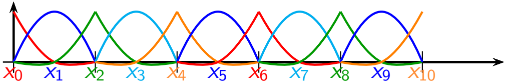


$$\tilde{A}_h^k = \frac{1}{h_k}\begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix}$$

$$i = \theta(k,\alpha) = 2(k-1) + \alpha, \qquad \alpha = 0, 1, 2, \qquad h_k = 2h$$

k=3

$$\tilde{A}_h = \frac{1}{2h}\begin{pmatrix}
\frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

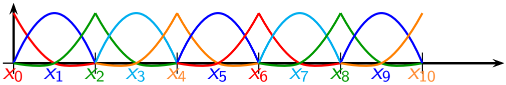


$$\tilde{A}_h^k = \frac{1}{h_k}\begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix}$$

$$i = \theta(k, \alpha) = 2(k-1) + \alpha, \qquad \alpha = 0, 1, 2, \qquad h_k = 2h$$

k=4

$$\tilde{A}_h = \frac{1}{2h}\begin{pmatrix}
\frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$



$$\tilde{A}_h^k = \frac{1}{h_k}\begin{pmatrix} \frac{7}{3} & -\frac{8}{3} & \frac{1}{3} \\ -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\ \frac{1}{3} & -\frac{8}{3} & \frac{7}{3} \end{pmatrix}$$

$$i = \theta(k,\alpha) = 2(k-1)+\alpha, \qquad \alpha = 0,1,2, \qquad h_k = 2h$$

k=5

$$\tilde{A}_h = \frac{1}{2h}\begin{pmatrix}
\frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{7}{3}
\end{pmatrix}$$

$$\tilde{A}_h \mathbf{u} = \tilde{\mathbf{F}}_h$$

or

$$\frac{1}{2h}\begin{pmatrix}
\frac{7}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{14}{3} & -\frac{8}{3} & \frac{1}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{8}{3} & \frac{16}{3} & -\frac{8}{3} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & -\frac{8}{3} & \frac{7}{3}
\end{pmatrix}
\begin{pmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \\ u_{10} \end{pmatrix}
=
\begin{pmatrix} F_0 \\ F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \\ F_9 \\ F_{10} \end{pmatrix}$$