Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4220 Numerical Solution of Partial Differential Equations Using Element Methods
Høst 2012

Exercise set 3

1 Quarteroni: Section 3.7: Exercise 4 and 5.

2 Write a MATLAB program for solving the Helmholtz problem

$$-u_{xx} + \sigma u = f(x), \qquad 0 < x < 1, \qquad u(0) = u(1) = 0.$$

or, using the weak formulation

$$\text{find } u \in H_0^1(0,1) \text{ s.t. } \int_0^1 u_x v_x dx + \sigma \int_0^1 uv dx = \int_0^1 fv dx, \text{ for all } v \in H_0^1(0,1) \quad (1)$$

by the finite element method on X_h^2, using the algorithm outlined in the supplementary note.

To test you code, let $\sigma = 1$, $f = \sin(\pi x)$ in which case $u(x) = \sin(\pi x)/(1+\pi^2)$.

Use for example $[0, 0.1, 0.25, 0.3, 0.4, 0.45, 0.5, 0.55, 0.6, 0.7, 0.8, 0.9, 1]$ for the partition of the elements. (That is, the first element is $[0, 0.1]$, and there is an extra node in the middle).

As already pointed out in Exercise 2.4, the discrete problem can be written as

$$(A + \sigma M)\mathbf{u} = \mathbf{b}. \quad (2)$$

So the task is to set up the matrices A and M and the load vector $\mathbf{b}$, and solve the system. What you have to do is described in the following:

a) Preliminaries:
Set up the element matrices A_h^K and M_h^K, corresponding to contribution from element K to the first and second integrals of (1) resp.

b) Write a function computing integrals by the following quadrature formula:

$$\int_0^1 g(x)dx \approx \frac{1}{2}(g(c_1) + g(c_2)), \qquad c_{1,2} = \frac{1}{2} \pm \frac{\sqrt{3}}{6}.$$

This will be used for to approximate the contribution from an element to the load vector.

c) Assemble the prototype matrices $\tilde{A}_h$ and $\tilde{M}_h$ as well as the load vector $\tilde{\mathbf{b}}$.

d) Remove the rows and columns corresponding to the boundary conditions.

e) Solve (2), and plot the solution

f) Change the boundary conditions to $u(0) = 1$ and $u_x(1) = 2$. Which changes has to be done in the code?