Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4220 Numerical Solution of Partial Differential Equations Using Element Methods
Fall 2012

Exercise set 5

1 Given the equation:

$$\begin{aligned} u_t &= u_{xx} + \beta u, \qquad 0 < x < 1 \\ \frac{\partial u}{\partial n}(0,t) &= 0, \quad u(1,t) = 0, \\ u(x,0) &= \cos\left(\frac{\pi}{2}x\right), \end{aligned}$$

and β is some constant.

a) Derive the exact solution for the equation.

b) Set up the weak formulation of the problem.

c) Write a MATLAB code to solve this problem. In space, use $V_h = X_h^1$ and a uniform grid. In time, try all three schemes: Forward and backward Euler, as well as Crank-Nicolson. Experiment with different stepsizes, and compare your numerical results with the exact solution.

2 Quarteroni Chapter 5, Exercise 2.
In b), no convergence analysis is required.

3 For those of you who have taken the course Numerical Mathematics or something equivalent:

Write down the set of fully discrete equations in the case of solving the semidiscretized system

$$M_h \dot{\mathbf{u}}(t) + A_h \mathbf{u}(t) = \mathbf{f}(t)$$

(Q: p.121, last line), by

a) A second order Adams-Bashforth scheme
b) A second order Adams-Moulton scheme
c) A second order Backward-Differentiation scheme

4 Problem 1-6 in the note *Spectra of the continuous and discrete Laplace operator* by Einar Rønquist.