



Norwegian University of Science  
and Technology  
Department of Mathematical  
Sciences

TMA4220 Numerical  
Solution of Partial  
Differential Equations  
Using Element Methods  
Fall 2013

**Project 5**

1 Given the equation:

$$\begin{aligned}u_t &= u_{xx} + \beta u, & 0 < x < 1 \\ \frac{\partial u}{\partial n}(0, t) &= 0, & u(1, t) = 0, \\ u(x, 0) &= \cos\left(\frac{\pi}{2}x\right),\end{aligned}$$

and  $\beta$  is some constant.

- a) Derive the exact solution for the equation.
- b) Set up the weak formulation of the problem.
- c) Write a MATLAB code to solve this problem. In space, use  $V_h = X_h^1$  and a uniform grid. If time, try all three schemes: Forward and backward Euler, as well as Crank-Nicolson. Experiment with different stepsizes, and compare your numerical results with the exact solution.

2 Quarteroni Chapter 5, Exercise 2.

In b), no convergence analysis is required.

3 For those of you who have taken the course Numerical Mathematics or something equivalent:

Write down the set of fully discrete equations in the case of solving the semidiscretized system

$$M_h \dot{\mathbf{u}}(t) + A_h \mathbf{u}(t) = \mathbf{f}(t)$$

(Q: p.121, last line), by

- a) A second order Adams-Bashforth scheme
- b) A second order Adams-Moulton scheme
- c) A second order Backward-Differentiation scheme

4 Problem 1-6 in the note *Spectra of the continuous and discrete Laplace operator* by Einar Rønquist.