1. Can we cover the 2-dimensional plane with a countable collection of circles?

2. Let $\{A_n\}$ be a sequence of open, dense subsets of a complete metric space (X, d). Prove that the intersection $\cap A_n$ is dense in X.

3. Use the Baire's category theorem to show the following: Any Hamel basis of an infinite-dimensional Banach space must contain uncountably many basis vectors. (*Hint*: Assume the fact that every finite-dimensional subspace is closed. Now, suppose that $B = \{x_n\}_{n=1}^{\infty}$ is a Hamel basis, and set $E_n = \text{span}\{x_1, \ldots, x_n\}$.)