

1. (Mazur's lemma) Let (x_n) be a weakly convergent sequence in a normed space X , say, $x_n \xrightarrow{w} x_0$. Show that $x_0 \in \overline{\text{co}}(\{x_n\}_{n \geq 1})$. Moreover, $\{x_0\} = \bigcap_{j \geq 1} K_j$, where $K_j = \overline{\text{co}}(\{x_n\}_{n \geq j})$. (*Hint*: Use the second geometric form of the Hahn-Banach theorem.)