1. Show that a sublinear function satisfies $p(0) = 0$ and $p(-x) \geq -p(x)$.

2. Let X be a real vector space and let $p : X \to \mathbb{R}$ be a sublinear function. Show that for any $x_0 \in X$ there exists a linear functional l on X such that $l(x_0) = p(x_0)$ and $l(x) \leq p(x)$ for all $x \in X$.

3. Show that in the Hahn-Banach theorem for real vector spaces, the conditions on p can be slightly relaxed as follows: $p(\alpha x + \beta y) \leq \alpha p(x) + \beta p(y)$ for all $x, y \in X$, and $\alpha, \beta \in [0, 1]$ with $\alpha + \beta = 1$.

4. Let X be a real normed space and let X_0 be a linear subspace of X. Suppose that $T_0 : X_0 \to \mathbb{R}^n$ is a bounded linear operator. Prove that there exists a bounded linear extension of T_0 on the whole space X.