

1. Let T be a positive operator on a Hilbert space H . Prove that $\ker T = \{x \in H \mid \langle Tx, x \rangle = 0\}$.
2. Show that $(B(H)_{\text{sa}}, \leq)$ is a partially ordered set.
3. Let $0 \leq A \leq B$ as operators on a Hilbert space. Show that $\|A\| \leq \|B\|$.
4. Show that the product of two positive operators is not necessarily positive.