

1. Let T be a bounded self-adjoint operator on a Hilbert space, and let $\lambda \in \rho(T)$. Show that

$$\|(T - \lambda I)^{-1}\| = \frac{1}{\text{dist}(\lambda, \sigma(T))},$$

where $\text{dist}(\lambda, \sigma(T)) = \inf\{|\lambda - \mu|; \mu \in \sigma(T)\}$.