

1. Prove that a hyperplane  $[f = \alpha]$  in a normed space  $X$  is closed if and only if  $f \in X'$ .
2. Prove that a hyperplane  $[f = \alpha]$  in a normed space  $X$  is dense if and only if  $f \notin X'$ . (*Hint:* First, let  $\alpha = 0$ . Then consider a sequence  $(x_n)$  such that  $x_n \rightarrow 0$  and  $|f(x_n)| \geq \epsilon$  for some  $\epsilon > 0$ . Now for any given  $x \in X$ , consider the sequence  $z_n = x - (f(x)/f(x_n))x_n$  .)