

# Kovarians og Korrelasjon

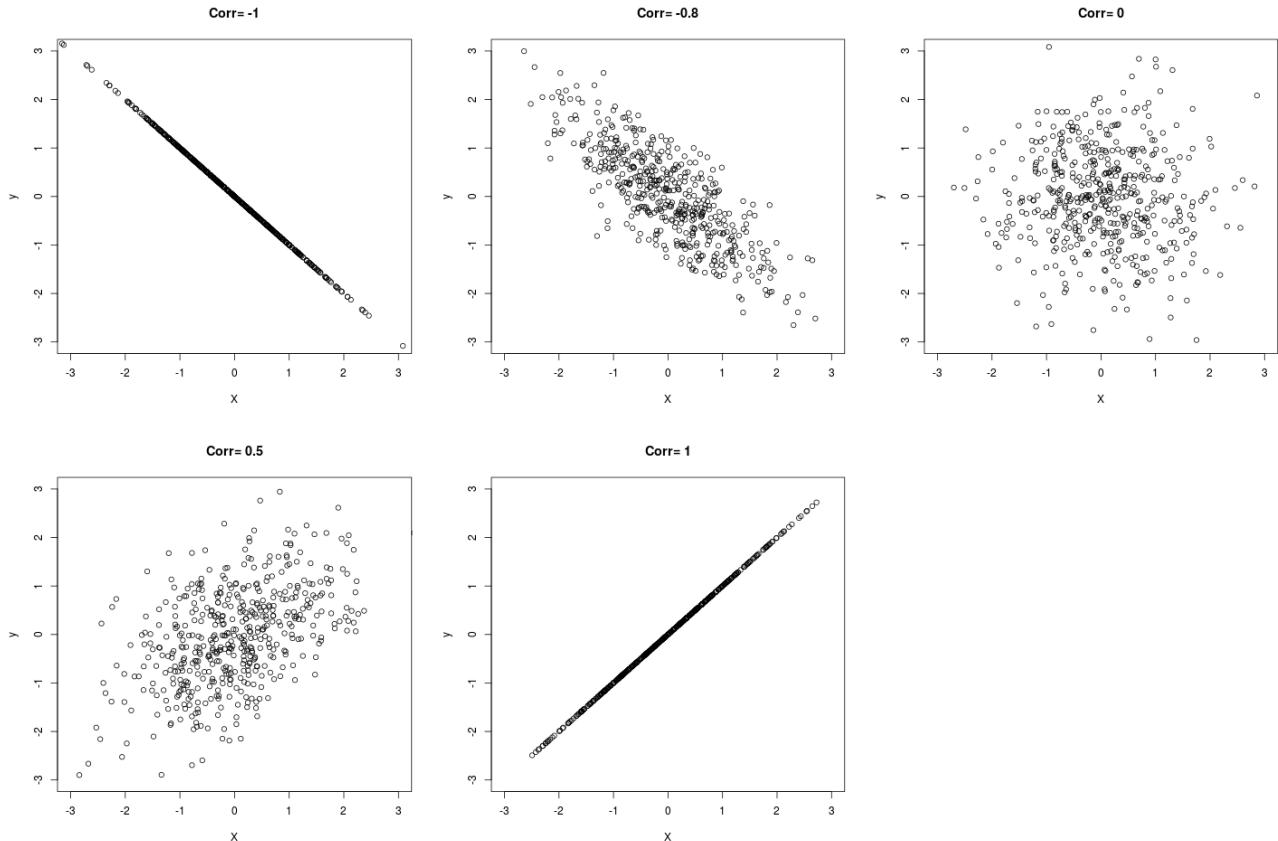
Kovarians:

$$\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] = E[XY] - E[X]E[Y]$$

Korrelasjon

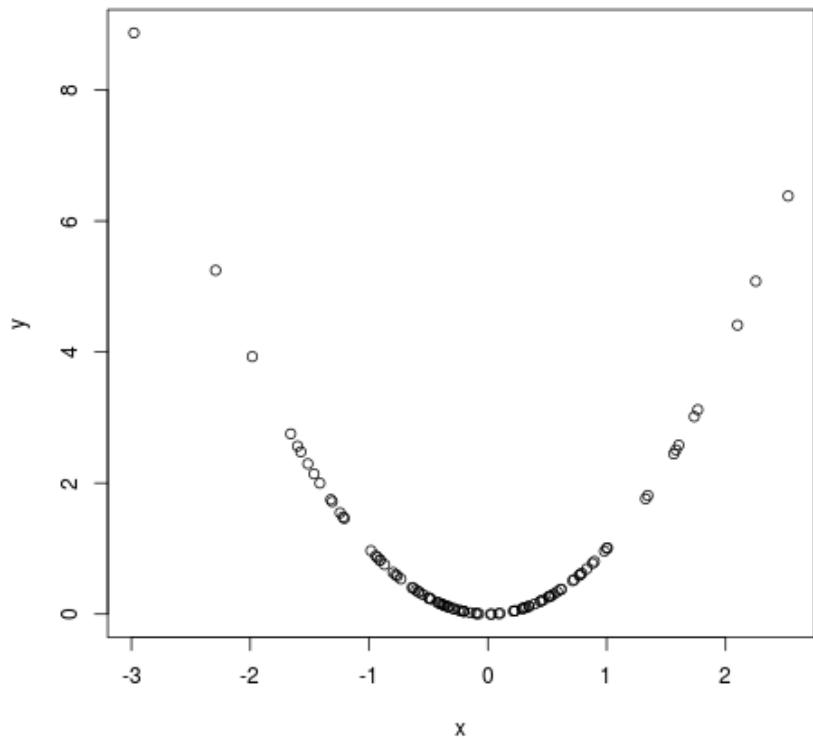
$$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}}$$

# Korrelasjon



# Korrelasjon

**Corr= -0.05**



# Bernoulliprosess

En Bernoulliprosess har følgende egenskaper:

- Består av  $n$  forsøk
- Hvert forsøk resulterer i suksess eller fiasko
- Suksess-sannsynligheten  $p$  er konstant
- Forsøkene er uavhengige

# Bernoullifordeling

$X$  har en Bernoullifordeling med suksess-sannsynlighet  $p$  hvis  $X \in \{0, 1\}$  og  $X$  har sannsynlighetsfordeling  $f$  gitt ved

$$f(x) = \begin{cases} p; & x = 1; \\ 1 - p; & x = 0 : \end{cases}$$

Vi skriver  $X \sim \text{Bernoulli}(p)$ .

# Binomisk fordeling

Antall suksess,  $X$ , i en Bernoulliprosess med  $n$  forsøk og suksess-sannsynlighet  $p$  har en binomisk fordeling

$$P(X = x) = f(x) = \binom{n}{x} p^x (1 - p)^{n-x}; x = 0, 1, 2, \dots, n.$$

Vi skriver  $X \sim \text{Binomial}(n; p)$ .

# Binomisk fordeling

$X \sim \text{Binomial}(n; p)$  er koblet til Bernoullifordelingen ved at  $X$  kan skrives som summen av  $n$  uavhengige stokastiske variabler,

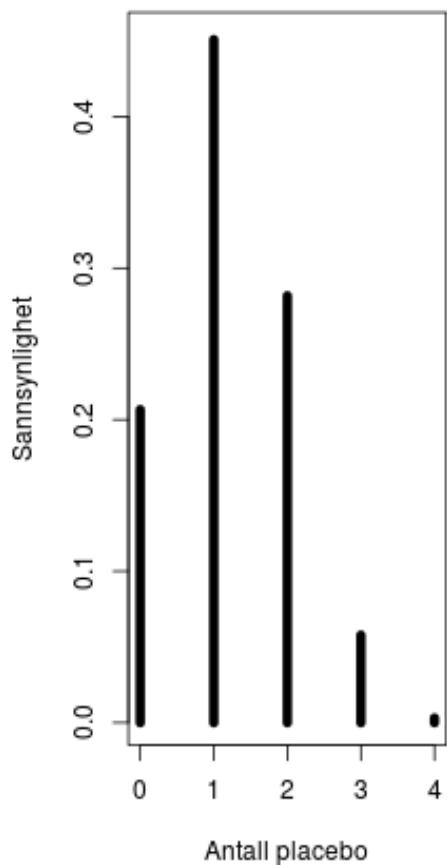
$$X = I_1 + I_2 + \cdots + I_n$$

der  $I_i \sim \text{Bernoulli}(p)$  for  $i = 1, 2, \dots, n$ . Dette kan brukes til å vise at

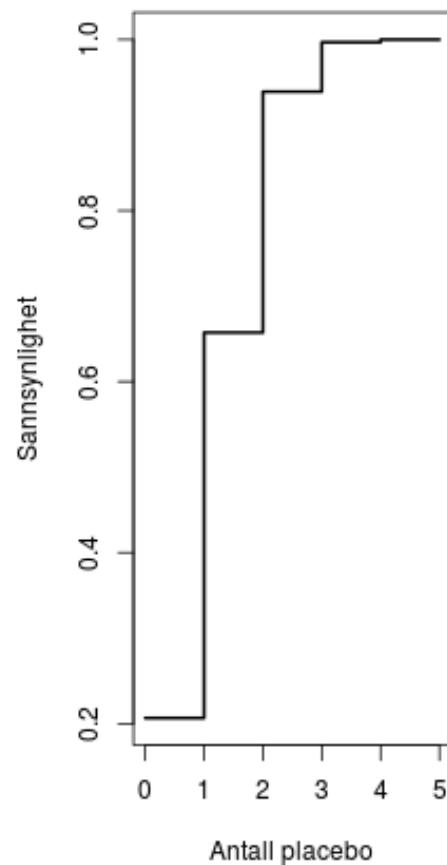
$$E[X] = np \text{ og } Var[X] = np(1 - p) :$$

# Hypergeometrisk Fordeling - Eksempel 1

Eks1: Punktsannsynlighet



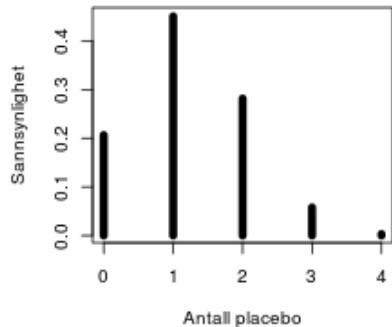
Eks1: Kumulativ sannsynlighet



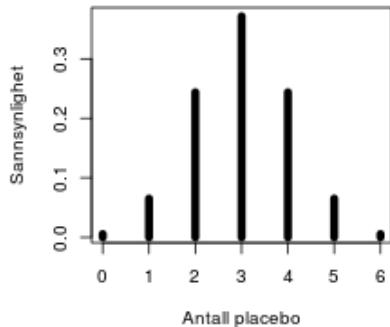
# Hypergeometrisk Fordeling - Eksempel 2

## Sannsynlighetsfordeling

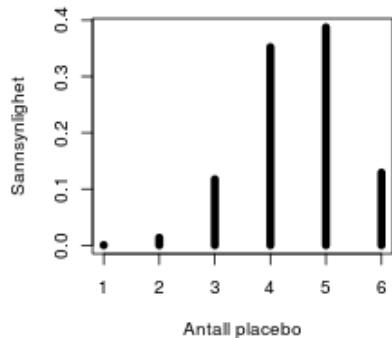
Eks2: n= 4



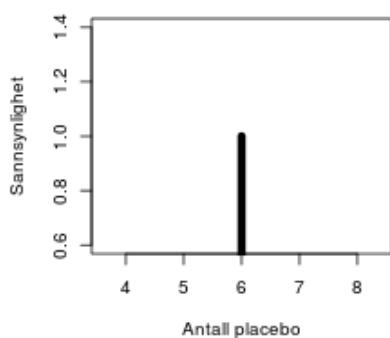
Eks2: n= 10



Eks2: n= 15



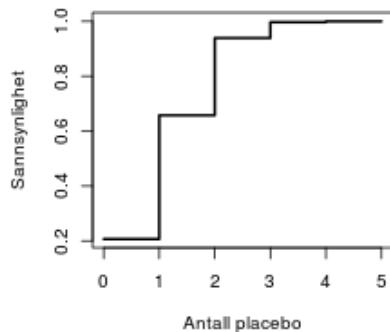
Eks2: n= 20



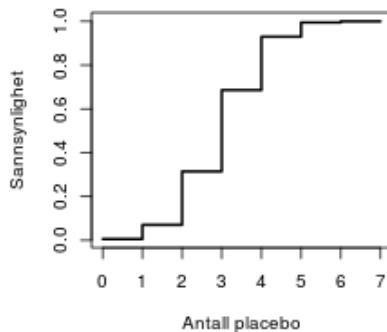
# Hypergeometrisk Fordeling - Eksempel 2

## Kumulativ fordeling

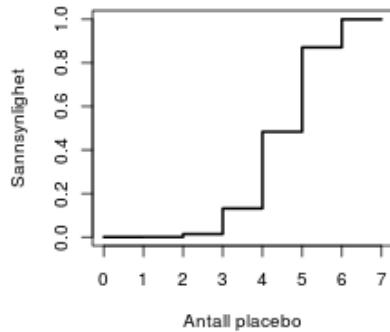
Eks2: n= 4



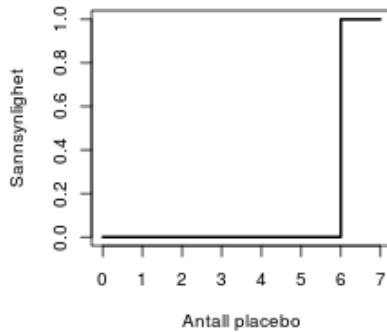
Eks2: n= 10



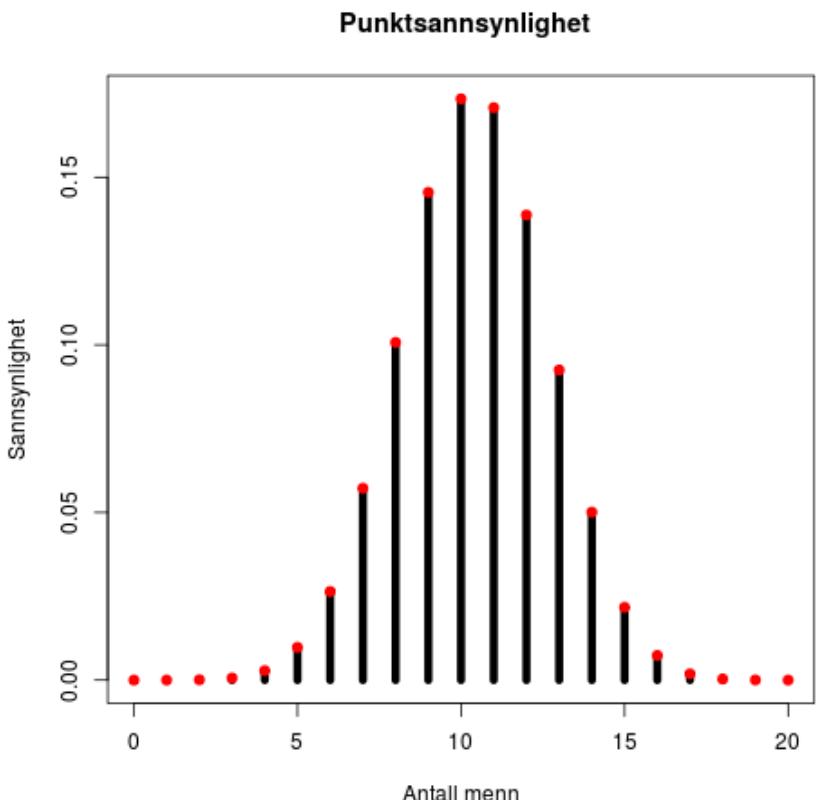
Eks2: n= 15



Eks2: n= 20



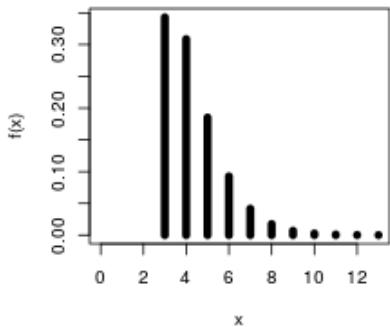
# Hypergeometrisk Fordeling - Eksempel 4



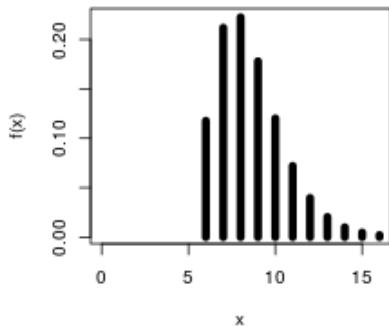
# Negativ-binomisk Fordeling - Eksempel 1

$k$  parameter

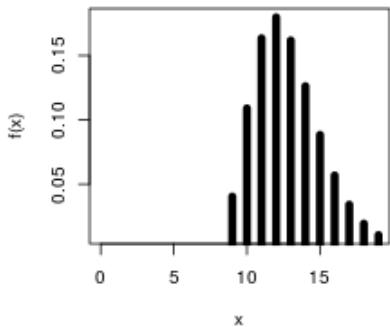
$k = 3, p = 0.7$



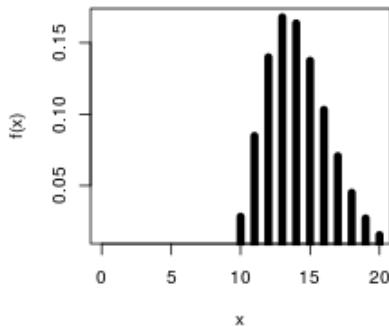
$k = 6, p = 0.7$



$k = 9, p = 0.7$



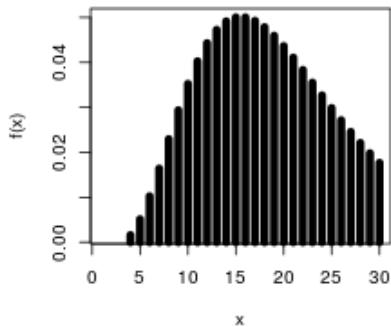
$k = 10, p = 0.7$



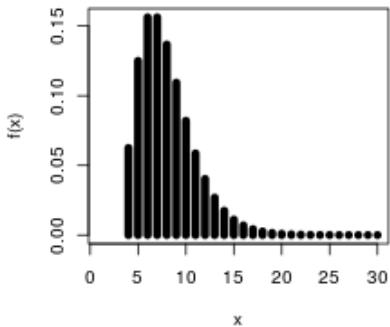
# Negativ-binomisk Fordeling - Eksempel 1

$p$  parameter

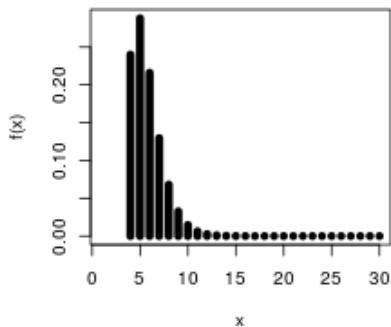
$k = 4, p = 0.2$



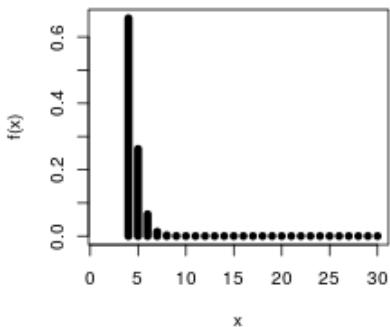
$k = 4, p = 0.5$



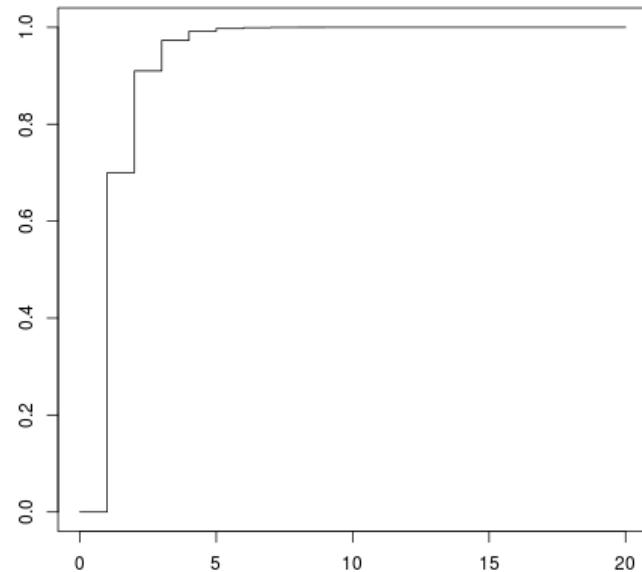
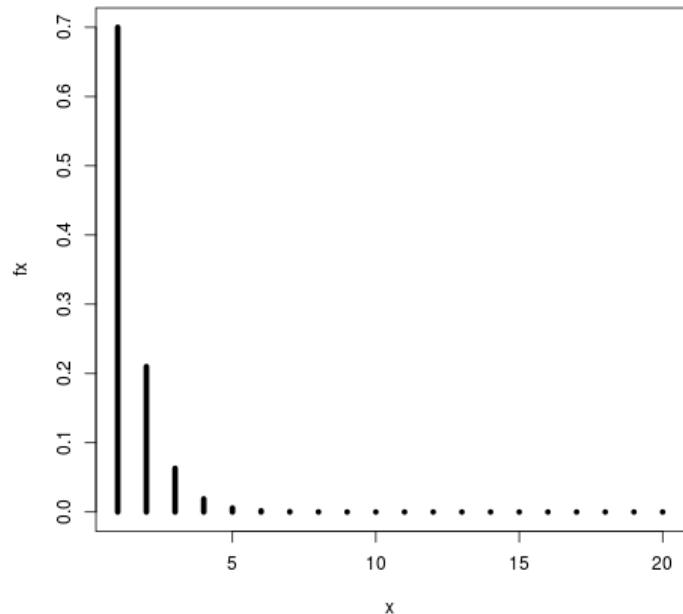
$k = 4, p = 0.7$



$k = 4, p = 0.9$



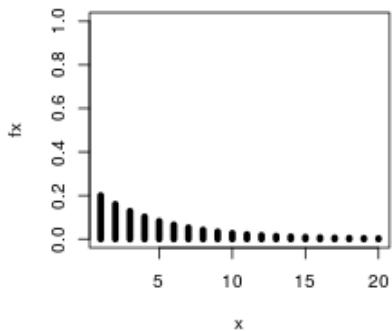
# Geometrisk Fordeling - Eksempel 1



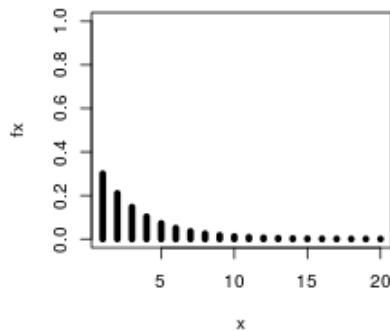
# Geometrisk Fordeling - Eksempel 1

$p$  parameter

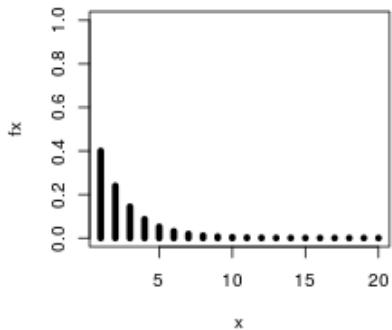
$p = 0.2$



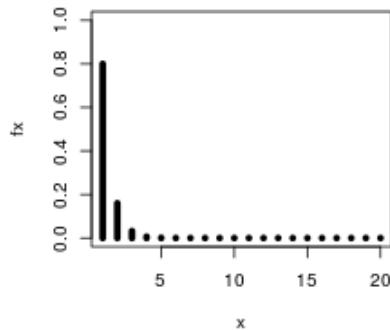
$p = 0.3$



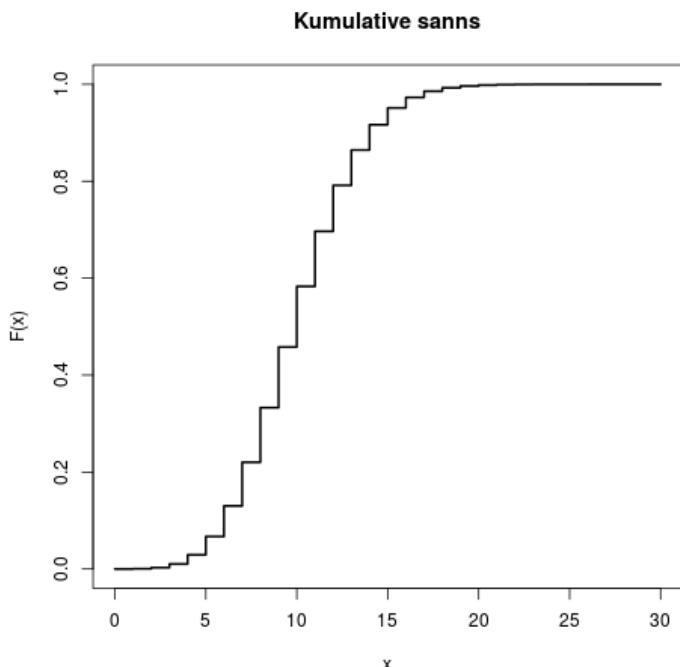
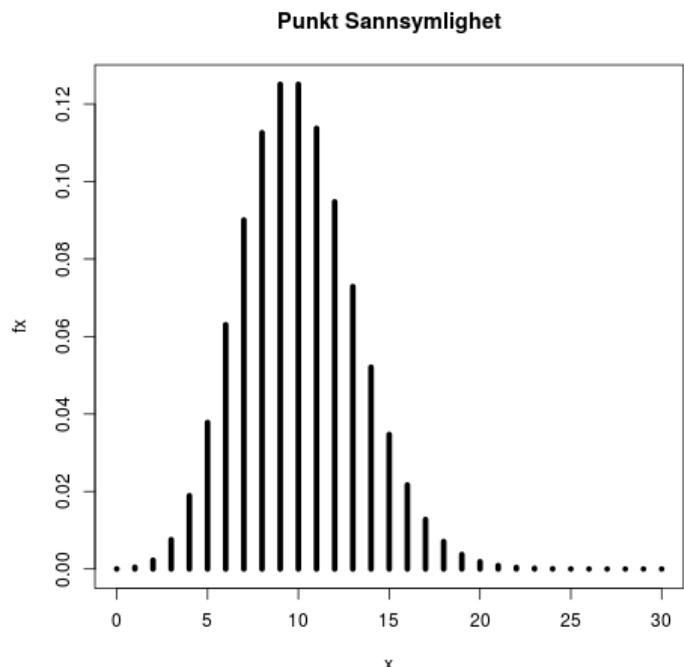
$p = 0.4$



$p = 0.8$



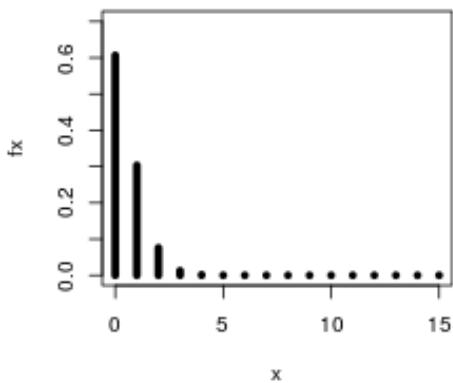
# Poisson Fordeling - Eksempel 1



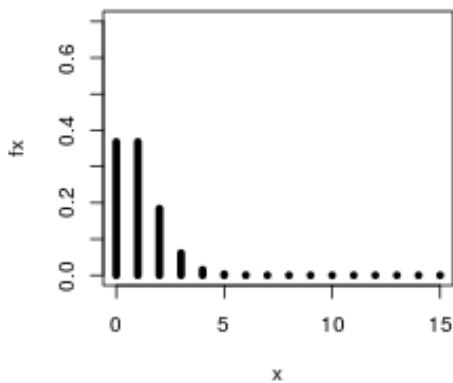
# Poisson Fordeling

Parameter  $\mu$

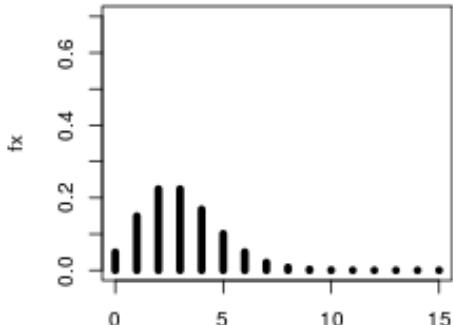
$\mu = 0.5$



$\mu = 1$



$\mu = 3$



$\mu = 5$

