TMA4250 Spatial Statistics
Assignment 2: Event Random Fields

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Introduction

This assignment contains problems related to Event Random Fields or more specificly Point Random Fields. The R-package should be used in solving the problems and relevant functions can be found in the R library `spatial` which can be loaded by the instruction `library(spatial)`

Problem 1: Poisson Random Field (PRF)

Consider the event RF $\{X_i \in \mathcal{D} \subset \mathcal{R}^2 \; ; \; i = 1, \ldots, N\}$ with $\mathcal{D} : [[0,1],[0,1]]$. Assume that the RF is an inhomogenuous Poisson RF with intensity function

$$\lambda(x_1, x_2; \mu, \alpha, \beta) = \mu \exp\{-(\alpha x_1 + \beta x_2)\} \; ; \; \boldsymbol{x} = (x_1, x_2) \in \mathcal{D}$$

where $\boldsymbol{\theta} = (\mu, \alpha, \beta)$ are model parameters.

a) The number of points in $\mathcal{D}$, N, is a random variable. Develop expressions for $\mathrm{E}\{N\}$ and $\mathrm{Var}\{N\}$

b) Set the model parameters to $\theta = (200, 2, 1)$ in the intensity function.
Generate realisations of the PRF conditional on the number of points being $n = 20$, 50 and 200 respectively.
Display the results and comment on them. Use the option `par(pty="s")` to display the points on a square.

c) Set the model parameters to $\theta = (200, 2, 1)$ in the intensity function.
Generate 10 realisations of the PRF.
Display the results and comment on them.

d) Choose one of the realisations in **c)**.
Specify the likelihood model for the parameters $\boldsymbol{\theta} = (\mu, \alpha, \beta)$ given the points in the chosen realisation. Develope an expression for the maximum likelihood

estimate of $\boldsymbol{\theta}$. Compute the numerical values of the maximum likelihood estimate of $\boldsymbol{\theta}$ given the points in the chosen realisation. Compare the estimates of $\boldsymbol{\theta}$ with the true value, and display the estimated and true intensity functions. Comment on the results. Use the function `image` to display the intensity function.

Problem 2: Neymann-Scott Random Field (N-SRF)

Consider an event RF $\{X_i \in \mathcal{D} \subset \mathcal{R}^2 \; ; \, i = 1, \ldots, N\}$ with $\mathcal{D} : [[0,1],[0,1]]$. Let the N-SRF be defined by the mother-model being a Poisson RF model and the child-model being numbers from Poisson pdf and intensity from uncorrelated bi-Gaussian pdf. These model assumptions defined the so called Thomas RF.

a) Vary the set of model parameters and generate realisations of the N-SRF. Pay particular attention to boundary-problems in the simulation algorithms. Explore the possibilities for modelling clustered event RFs.
Display the results and comment on them.

b) Use the function `Kfn` in the `R` library `spatial` to estimate the L-function for five extreme realisations from **a)**.

The theoretical L-function for Thomas RF is

$$K(t) = \pi t^2 + \lambda^{-1}(1 - \exp\{-t^2/4\sigma^2\})$$

where λ is the intensity of the mother-model and σ^2 is the variance of the intensity function of the child-model. Display the estimated and the theoretical L-function for the five realisations.
Comment on the results.

Problem 3: Strauss Random Field (SRF)

Consider an event RF $\{X_i \in \mathcal{D} \subset \mathcal{R}^2 \; ; \, i = 1, \ldots, N\}$ with $\mathcal{D} : [[0,1],[0.1]]$. Assume that the pdf, given the number of points $N = n$, related to this RF is:

$$f(x_1, \ldots, x_n | n; \theta) = \text{const} \times \exp\{-\sum_{i=1}^{n}\sum_{j=1}^{n} \varphi(\tau_{ij}; \theta)\}$$

with $\tau_{ij} = |x_i - x_j|$ being the euclidean distance between x_i and x_j, and

$$\varphi(\tau; c, d) = \begin{cases} c & \text{if } \tau < d \\ 0 & \text{else} \end{cases}$$

This RF is termed a Strauss RF.

a) Some special versions of SRFs are identical to Binomial RF. Specify the parameter values which defines these special cases. Which additional assumptions are needed in order to define a Poisson RF with intensity λ?

b) Set $n = 50$ and $d = 0.1$ and generate realisations of the SRF with $c = 0.01$, 1 and 100 respectively. Document that the algorithm has converged.
Display the results and comment on them.

Use the function `ppregion` to initialise the point RF region.

Problem 4: Analysis of Point Pattern

Consider three real data point patterns in the R library MASS:

- biological cell data, available at `cells.dat`
- redwood tree data, available at `redwood.dat`
- pine tree data, available at `pines.dat`

Use the command `data<-ppinit("file.dat")` to load the data files.

a) Display the three point patterns and comment on them.

b) Compute the L-function for each of the point pattern.
Display the results and comment on them.

Compare the computed L-functions for each of the point patterns with the theoretical L-functions for a Poisson RF.
Display the results and comment on them.

Take the number of points in each point pattern into account. Generate 100 realisations of corresponding Poisson RF and compute 100 L-functions. Estimate expected L-function with associated 0.90 envelopes. Use the L-function results to test informally whether each of the point patterns could come from an underlying Poisson RF.
Display the results and comment on them.

c) Perform the same procedure as in **b)** on the pdf of R, the distance from a given point to its closest neighbour.