Chess example

a) $\{X_n\}_{n=0}^{\infty}$ is a Markov chain as the probability of which state the chain reaches in the next step only depends on the current state. Since all states communicate the Markov chain is irreducible. Since there is only one equivalence class and periodicity is a class property we only need to determine the ~~prop~~ period for one state. Since $P_{00} > 0$ the chain is aperodic as it is possible to return to 0 in 1, 2, 3, ... steps.

b)

$$P(X_3 = 0, X_2 = 0, X_1 = 0 \mid X_0 = 0) = 0.6^3 = 0.216$$

$$P(X_3 = 0 \mid X_0 = 0) = 0.6^3 + 6 \cdot 0.6 \cdot 0.3 \cdot 0.1 = 0.324$$

c) See next page

d) As discussed with Haakon

The expected number of games played before one of the players is in lead by three games is given by the sum of row 3:

$$\sum_{j=-2}^{2} s_{0j} = 0.36 + 1.43 + 4.64 + 4.29 + 3.21 = 13.93 \approx 14 \text{ games}$$

The probability that the chain never enters state -1 given starts in state 0 is given by.

$$1 - f_{0,-1} = 1 - \frac{s_{0,-1} - 0}{s_{-1,-1}} = 1 - \frac{1.43}{4.40} = 1 - 0.325 = \underline{\underline{0.675}}$$

c)

A : be the event that 2 happens before -2

Let $u_i = P(A | X_0 = i)$

We are interested in u_0.

$u_{-2} = 0$

$u_{-1} = \cancel{0.1 u_{-2}} + 0.6 u_{-1} + 0.3 u_0 \Rightarrow u_{-1} = \frac{3}{4} u_0$

$u_0 = 0.1 \cdot u_{-1} + 0.6 u_0 + 0.3 u_1$

$u_1 = 0.1 u_0 + 0.6 u_2 + 0.3 u_2 \Rightarrow u_1 = \underbrace{\frac{0.1 u_0 + 0.3}{0.4}}_{\frac{1}{4} u_0 + \frac{3}{4}}$

$u_2 = 1$

$\Rightarrow u_0 = 0.1 \cdot \frac{3}{4} u_0 + 0.6 u_0 + 0.3 \cdot \frac{0.1 u_0 + 0.3}{0.4}$

$= \frac{3}{40} u_0 + \frac{6}{10} u_0 + \frac{3}{40} u_0 + \frac{9}{40}$

$= \frac{30}{40} u_0 + \frac{9}{40}$

$\Rightarrow u_0 = \frac{9}{10}$

The probability that Magnus is the first player to be in lead with two games is 90%