TMA 4275 Lifetime Analysis Spring 2017
Homework 5

Problem 1 (Exam June 2007, modified)

The following are the mileages at which 10 military personnel carriers failed in service. There were no censored observations.

271	320	629	706	777
1182	1463	1603	1484	2355

You are supposed to solve items (a)-(d) exercise "by hand", but you may of course check your results using MINITAB.

a) Make the TTT-plot for the data and comment on the suitability of the exponential model for it.

b) Carry out a test of exponentiality against the IFR alternative at 5% level of significance, using the Barlow-Proschan test.

c) Assuming that the mileages at failure are exponentially distributed with expected value θ, estimate θ and also obtain a 95% confidence interval for θ.

d) Compute the maximum value of the log likelihood function using the exponential model.

e) Assume now the data are Weibull-distributed with shape parameter α and scale parameter θ. Use MINITAB to estimate the parameters, and estimate also MTTF using the Weibull model. Compare the estimated MTTF with the one obtained from the exponential model and comment.

f) Read off the maximum value of the log likelihood also for the Weibull model. Compare to the one computed for the exponential model. How can you use these values to obtain a new test of the situation considered in item (b)?

Problem 2

Assume that you have determined the lifetimes for a total of 12 identical items and obtained the following results (given in hours):

10.2, 89.6, 54.0, 96.0, 23.3, 30.4, 41.2, 0.8, 73.2, 3.6, 28.0, 31.6

Assume that the data is a random sample (non-censored) from an exponential distribution with unknown *hazard rate* λ, i.e., with density function

$$f(t; \lambda) = \lambda e^{-\lambda t}$$

for $t > 0$, where $\lambda > 0$ now is the unknown parameter.

a) Find an estimate for λ.

b) Determine a 95% confidence interval for λ by using

- standard interval
- standard interval for positive parameters
- likelihood method

Problem 3

Suppose that 5 components were put on test. All that is known is that they all failed between $t = 1$ and $t = 4$. You should solve the exercise both using MINITAB and by doing the computations on paper.

a) If their lifetime is exponential, what is the estimated mean time to failure, θ?

b) What is the estimated standard deviation of the maximum likelihood estimator $\hat{\theta}$ of θ (i.e. the standard error of $\hat{\theta}$)?

c) Find a 95% confidence interval for θ by any suitable method.

Problem 4 – Estimation and testing in the gamma distribution

Assume that the lifetime T is gamma distributed with shape parameter 2, so that

$$f(t; \lambda) = \lambda^2 t e^{-\lambda t}$$

for $t > 0$, where $\lambda > 0$ is the unknown parameter. We have n independent observations $t_1, \ldots, t_n$ of T (no censoring).

a) Find the MLE $\hat{\lambda}$ for λ. What are the properties of this estimator?

b) What is the estimate for λ when $n = 10$, $\sum t_i = 180$ (months)? Also find an estimate for the standard deviation of $\hat{\lambda}$ (i.e., standard error).

Remark: MINITAB does not include statistical inference for the gamma distribution, so you need to compute this by yourself.

c) Perform a test based on the log likelihood for the hypotheses

$$H_0 : \ \lambda = 0.25 \text{ versus } H_1 : \ \lambda \neq 0.25$$

What is the conclusion if the significance level is 5%?

d) Make a confidence interval for λ using the loglikelihood method (the "1.92 confidence interval").