TMA4275 Life time analysis
Exercise 2, Spring 2021

Problem 1: Problem 2.3 in ABG

Problem 2: Problem 2.1 in ABG.

Problem 3: Problem 2.4 in ABG.

Problem 4:

(a) Let $X_1, X_2, \ldots$ be independent stochastic variables with $\mathrm{E}[X_i] = 0$ for $n \geq 1$. Let $M_0 = 0$ and $M_n = X_1 + \ldots + X_n$ for $n \geq 1$. Then show that M_n is a (mean-zero) martingale.

(b) Let $X_1, X_2, \ldots$ be independent stochastic variables with $\mathrm{E}[X_i] = \mu$ for $n \geq 1$. Let $S_0 = 0$ and $S_n = X_1 + \ldots + X_n$ for $n \geq 1$. Then find $\mathrm{E}[S_n|\mathcal{F}_n]$, where $\mathcal{F}_n$ contains all information about the X process up to and including time n. Is S_n a (mean-zero) martingale? Can you find a simple transformation of S_n which is a zero-mean martingale?

(c) Let $X_1, X_2, \ldots$ be independent stochastic variables with $\mathrm{E}[X_i] = 1$ for $n \geq 1$. Let $M_0 = 1$ and $M_n = X_1 \cdot \ldots \cdot X_n$ for $n \geq 1$. Then show that M_n is a martingale. What is $\mathrm{E}[M_n]$?

Problem 5: Problem 2.6 in ABG.

Problem 6: Problem 2.7 in ABG.

Problem 7: Let $X_0 = 0$ and $X_n = U_1 + \ldots + U_n$ for $n = 1, 2, \ldots$, where $U_1, U_2, \ldots$ are independent and identically distributed stochastic variables with $\mathrm{E}[U_i] = \mu$.

Find the Doob decomposition of the process X. In particular identify the predictable and the innovation part of the X process.