TMA4275 Life time analysis
Exercise 4, Spring 2021

Problem 1: Problem 3.1 in ABG. Compute the estimates and make the plots by hand or by writing your own code in R, i.e. do not use available R functions to compute the Nelson-Aalen estimates.

Problem 2: Problem 3.2 in ABG.

Problem 3: Problem 3.3 in ABG.

Problem 4: Let $N_i = \{N_i(t); t \in [0, \tau]\}$ for $i = 1, \ldots, n$ be independent nonhomogenuous Poisson prosesses (NHPP), all with the same intensity function $\alpha(t)$. Moreover, let $A(t) = \int_0^t \alpha(s)ds$ and let $\mathcal{F}_t$ contain the information about all the N_i processes up to and including time t. Thus, we in particular have

- $N_i(t) - N_i(s) \sim \text{Poisson}(A(t) - A(s))$ when $s < t$, and
- $N(t) - N(s)$ is independent of $\mathcal{F}_s$ when $s < t$.

For each $i = 1, \ldots, n$ assume that the N_i process is observed only on an interval $[0, \tau_i]$, where $\tau_i \leq \tau$. Then define the aggregated observed process $N = \{N(t); t \in [0, \tau]\}$, where

$$N(t) = \sum_{i=1}^{n} N_i(\min\{t, \tau_i\}).$$

(a) Explain that $N_i(\min\{t, \tau_i\})$ is the <u>observed</u> number of observations in the process N_i at time t. Moreover, explain that $N_i(\min\{t, \tau_i\})$ has a multiplicative intensity model $\lambda_i(t) = \alpha(t)Y_i(t)$ and give an expression for $Y_i(t)$.

(b) Show that the aggregated observed process N also has a multiplicative intensity model $\lambda(t) = \alpha(t)Y(t)$, and specify an expression for $Y(t)$.

(b) Write down an expression for the Nelson-Aalen estimator for $\alpha(t)$ based on the observed process N.

(c) Suppose $n = 3, \tau_1 = 20, \tau_2 = 30, \tau_3 = 10$ and that the observed event times are 5, 12 and 17 for process N_1, 9 and 23 for process N_2 and 4 for process N_3.

Calculate the Nelson-Aalen estimator for $A(t)$ (by hand) and draw a graph.

Calculate also the estimate of the variance of the estimator and find a 95% confidence bounds for the Nelson-Aalen curve.

(d) Formulate a result of the asymptotic behaviour of the Nelson-Aalen estimator for $A(t)$ as the number of processes n goes to infinity. Consider in particular the case when $\tau_i = \tau$ for all $i = 1, \ldots, n$.