

Plan for this lecture

- ★ Anything to say about the project?
- ★ Recall some theory parts
 - partial likelihood function for a relative risk model
 - score test for a relative risk model
 - two-sample nonparametric log-rank test
- ★ Cox regression with one binary covariate
- ★ Score test for a Cox regression situation with one binary covariate
 - two-sample log-rank test

Partial likelihood function for a relative risk model

★ Situation:

- n individuals
- individual i has covariate vector $x_i(t)$
- individual i has hazard rate and intensity process

$$\begin{aligned}\alpha(t|x_i(t)) &= \alpha_0(t)r(\beta, x_i(t)) \\ \lambda_i(t) &= Y_i(t)\alpha_0(t)r(\beta, x_i(t))\end{aligned}$$

- $N_i(t), Y_i(t), N_{\bullet}(t), Y_{\bullet}(t), \lambda_{\bullet}(t)$

★ Partial likelihood

$$L(\beta) = \prod_j \left[\frac{r(\beta, x_{i_j}(T_j))}{\sum_{l=1}^n Y_l(T_j)r(\beta, x_l(T_j))} \right]$$

★ For Cox regression models:

$$r(\beta, x_i(t)) = \exp\{\beta^T x_i(t)\}$$

Score test for a relative risk model

★ Situation:

- n individuals
- individual i has covariate vector $x_i(t)$
- individual i has hazard rate and intensity process

$$\alpha(t|x_i(t)) = \alpha_0(t)r(\beta, x_i(t))$$

$$\lambda_i(t) = Y_i(t)\alpha_0(t)r(\beta, x_i(t))$$

- $N_i(t)$, $Y_i(t)$, $N_{\bullet}(t)$, $Y_{\bullet}(t)$, $\lambda_{\bullet}(t)$

★ Score test: $H_0 : \beta = \beta_0$

- test statistic

$$\chi_{SC}^2 = U(\beta_0)^T I(\beta_0)^{-1} U(\beta_0)$$

where χ_{SC}^2 is approximately χ^2 -distributed with q degrees of freedom when H_0 is true

- $U(\beta) = \nabla \ell(\beta)$
- $I(\beta) = -\nabla U(\beta) = -\nabla^2 \ell(\beta)$

Two-sample nonparametric log-rank test

- ★ Have two counting processes $N_1(t)$ and $N_2(t)$ with intensity processes

$$\lambda_1(t) = Y_1(t)\alpha_1(t)$$

$$\lambda_2(t) = Y_2(t)\alpha_2(t)$$

- ★ Log-rank test: $H_0 : \alpha_1(t) = \alpha_2(t), t \in [0, t_0]$

- test statistic

$$\chi_{LR}^2 = \frac{Z_1(t_0)^2}{V_{11}(t_0)}$$

where χ_{LR}^2 is approximately χ^2 distributed with 1 degree of freedom when H_0 is true

- $Z_1(t_0) = \int_0^{t_0} \frac{L(t)}{Y_1(t)} dN_1(t) - \int_0^{t_0} \frac{L(t)}{Y_2(t)} dN_2(t)$
- $V_{11}(t_0) = \int_0^{t_0} \frac{L^2(t)}{Y_1(t)Y_2(t)} dN_{\bullet}(t)$
- $L(t) = \frac{Y_1(t)Y_2(t)}{Y_{\bullet}(t)}$