Plan for this lecture

- Recall some theory parts
  - parametric counting process models — situation
  - likelihood function
  - maximum likelihood estimators
  - assymptotic properties
- Exponential regression

Parametric counting process models

- Situation
  - n individuals
  - individual i has hazard rate and intensity process

$$\alpha(t|x_i) = \alpha_0(t;\theta)r(\beta, x_i)$$
$$\lambda_i(t;\theta,\beta) = Y_i(t)\alpha_0(t;\theta)r(\beta, x_i)$$

  - aggregated intensity process

$$\lambda_\bullet(t;\theta,\beta) = \sum_{i=1}^{n} \lambda_i(t,\theta,\beta)$$

- Likelihood function

$$L(\theta,\beta) = \left[\prod_{i=1}^{n} \prod_{0<t\leq\tau} \lambda_i(t;\theta,\beta)^{\Delta N_i(t)}\right] \exp\left\{-\int_0^\tau \lambda_\bullet(t;\theta,\beta)dt\right\}$$

- Log-likelihood function

$$\ell(\theta,\beta) = \ln L(\theta,\beta)$$

Assymptotic properties

- The maximum likelihood estimators are

$$\left(\widehat{\theta}, \widehat{\beta}\right) = \underset{(\theta,\beta)}{\text{argmax}}\, \ell(\theta, \beta)$$

- Score function

$$U(\theta, \beta) = \nabla \ell(\theta, \beta)$$

- Information matrix

$$\mathbb{I}(\theta, \beta) = -\nabla^2 \ell(\theta, \beta)$$

- We have approximately that

$$\left(\widehat{\theta}, \widehat{\beta}\right) \sim \mathsf{N}\left(\left[\begin{array}{c} \theta \\ \beta \end{array}\right], \mathbb{I}\left(\widehat{\theta}, \widehat{\beta}\right)^{-1}\right)$$

where θ and β are the true values.