Plan for this lecture

- Recall some theory parts
  - likelihood function for general counting process
  - Laplace transform
  - formulas for gamma distribution with mean 1 and variance δ
- One situation with shared frailty models
  - recurrent events and the Poisson process
- Derive formulas from likelihood function for general counting process
- Use formulas for Laplace transform of gamma distribution

Formulas to be used

- Likelihood function for general counting process
  - (5.4) in ABG

$$L(\theta) = P_\theta(data) = \left[\prod_{i=1}^{n} \prod_{0<t\leq\tau} \lambda_i(t;\theta)^{\Delta N_i(t)}\right] \exp\left\{-\int_0^\tau \lambda_\bullet(t;\theta)dt\right\}$$

- Laplace transform

$$\mathcal{L}(c) = \mathrm{E}\left[e^{-cZ}\right]$$

$$\mathcal{L}^{(r)}(c) = (-1)^r \mathrm{E}\left[Z^r e^{-cZ}\right]$$

- Gamma distribution with mean 1 and variance δ

$$f_Z(z) = \frac{\left(\frac{1}{\delta}\right)^{\frac{1}{\delta}}}{\Gamma\left(\frac{1}{\delta}\right)} z^{\frac{1}{\delta}-1} \exp\left\{-\frac{z}{\delta}\right\}$$

$$\mathcal{L}(c) = (1+\delta c)^{-\frac{1}{\delta}}$$

$$\mathcal{L}^{(r)}(c) = (-1)^r \delta^{r-1} (1+\delta c)^{-\frac{1}{\delta}-r} \prod_{q=1}^{r-1}\left(\frac{1}{\delta}+q\right)$$