

Plan for the lecture

- ★ The Weibull distribution
 - hazard rate, survival function, cumulative distribution function, density function
- ★ The laws of total probability and double expectation survival times
 - discrete stochastic variables
 - continuous stochastic variables
 - conditioning on histories
- ★ A counting process example

Relations between $f(t)$, $F(t)$, $\alpha(t)$, $S(t)$ and $A(t)$

$$f(t) = F'(t)$$

$$F(t) = \int_0^t f(s) ds$$

$$S(t) = 1 - F(t)$$

$$f(t) = -S'(t)$$

$$A(t) = \int_0^t \alpha(s) ds$$

$$\alpha(t) = A'(t)$$

$$\alpha(t) = -\frac{d}{dt} \ln S(t)$$

$$S(t) = \exp\{-A(t)\}$$

Counting process and the intensity process

- ★ Counting process:

- consider particular type of event
- at most one event happens in short time interval $(t, t + dt]$
- $N(t)$: # of events from time 0 to time (including) time t
- increment: $dN(t) = N(t + dt) - N(t)$

- ★ Intensity process $\lambda(t)$:

$$\lambda(t)dt = P(dN(t) = 1|\text{past}) = E[dN(t)|\text{past}]$$

- ★ Corresponding martingale

$$M(t) = N(t) - \int_0^t \lambda(s)ds$$