

Plan for this lecture

- ★ Briefly recall some theory parts
 - additive regression models
 - situation
 - notation
 - estimators
- ★ Additive regression model with one binary covariate
 - derive formulas for the estimators
 - hypothesis test

Additive regression models — situation

- n individuals
- individual i has covariate vector $x_i(t)$
- individual i has hazard rate and intensity process

$$\begin{aligned}\alpha(t|x_i(t)) &= \alpha_0(t)r(\beta, x_i(t)) \\ \lambda_i(t) &= Y_i(t)\alpha_0(t)r(\beta, x_i(t))\end{aligned}$$

where

$$r(\beta, x_i(t)) = \beta_0(t) + \beta_1(t)x_{i1}(t) + \dots \beta_p(t)x_{ip}(t)$$

- $N_i(t)$, $Y_i(t)$, $N_{\bullet}(t)$, $Y_{\bullet}(t)$, $\lambda_{\bullet}(t)$
- ★ $B_q(t) = \int_0^t \beta_q(u)du$ for $q = 0, 1, \dots, p$

Additive regression models — notation and estimator

- ★ $\mathbb{N}(u) = [N_1(u) N_2(u) \cdots N_n(u)]^T$

- ★ $\mathbb{X}(u)$:

$$\mathbb{X}(u) = \begin{bmatrix} Y_1(u) & Y_1(u)x_{11}(u) & \cdots & Y_1(u)x_{1p}(u) \\ Y_2(u) & Y_2(u)x_{21}(u) & \cdots & Y_2(u)x_{2p}(u) \\ \vdots & \vdots & & \vdots \\ Y_n(u) & Y_n(u)x_{n1}(u) & \cdots & Y_n(u)x_{np}(u) \end{bmatrix}$$

- ★ $\mathbb{X}^-(u) = (\mathbb{X}(u)^T \mathbb{X}(u))^{-1} \mathbb{X}(u)^T$, $(p+1) \times n$ matrix

- ★ $J(u) = I(\mathbb{X}(u) \text{ has full rank})$, scalar

- ★ Estimator

$$\hat{\mathbb{B}}(t) = \begin{bmatrix} \hat{B}_0(t) \\ \hat{B}_1(t) \\ \vdots \\ \hat{B}_p(t) \end{bmatrix} = \int_0^t J(u) \mathbb{X}^-(u) d\mathbb{N}(u)$$

The TST-test

- ★ Assume situation as in additive regression model
- ★ Want to test

$$H_0 : \beta_q(t) = 0 \quad \text{for all } t \in [0, t_0]$$

- ★ Test statistic

$$U_{TST} = \frac{Z_q(t_0)}{\sqrt{V_{qq}(t_0)}} \approx N(0, 1) \quad \text{when } H_0 \text{ is true,}$$

where

$$\begin{aligned} Z_q(t_0) &= \int_0^{t_0} L_q(t) d\widehat{\mathbb{B}}_q(t) = \sum_{j: T_j \leq t_0} L_q(T_j) \Delta \mathbb{B}_q(T_j) \\ V_{qq}(t_0) &= \int_0^{t_0} L_q^2(t) d\widehat{\sigma}_{qq}(t) = \sum_{j: T_j \leq t_0} L_q^2(T_j) \Delta \widehat{\sigma}_{qq}(T_j) \\ L_q(t) &= \mathbb{K}_{qq}(t), \mathbb{K}(t) = \left\{ \text{diag} \left[(\mathbb{X}(t)^T \mathbb{X}(t))^{-1} \right] \right\}^{-1} \end{aligned}$$

Some results in our example situation

★ $\mathbb{X}(u)$:

$$\mathbb{X}(u) = \begin{bmatrix} Y_1(u) & Y_1(u)x_1 \\ Y_2(u) & Y_2(u)x_2 \\ \vdots & \vdots \\ Y_n(u) & Y_n(u)x_n \end{bmatrix}$$

Some results in our example situation

★ $\mathbb{X}(u)$:

$$\mathbb{X}(u) = \begin{bmatrix} Y_1(u) & Y_1(u)x_1 \\ Y_2(u) & Y_2(u)x_2 \\ \vdots & \vdots \\ Y_n(u) & Y_n(u)x_n \end{bmatrix}$$

★ $\mathbb{X}(u)^T \mathbb{X}(u)$:

$$\mathbb{X}(u)^T \mathbb{X}(u) = \begin{bmatrix} Y_{\bullet}(u) & Y^{(1)}(u) \\ Y^{(1)}(u) & Y^{(1)}(u) \end{bmatrix}$$

Some results in our example situation

★ $\mathbb{X}(u)$:

$$\mathbb{X}(u) = \begin{bmatrix} Y_1(u) & Y_1(u)x_1 \\ Y_2(u) & Y_2(u)x_2 \\ \vdots & \vdots \\ Y_n(u) & Y_n(u)x_n \end{bmatrix}$$

★ $\mathbb{X}(u)^T \mathbb{X}(u)$:

$$\mathbb{X}(u)^T \mathbb{X}(u) = \begin{bmatrix} Y_{\bullet}(u) & Y^{(1)}(u) \\ Y^{(1)}(u) & Y^{(1)}(u) \end{bmatrix}$$

★ $(\mathbb{X}(u)^T \mathbb{X}(u))^{-1}$:

$$(\mathbb{X}(u)^T \mathbb{X}(u))^{-1} = \begin{bmatrix} \frac{1}{Y^{(0)}(u)} & -\frac{1}{Y^{(0)}(u)} \\ -\frac{1}{Y^{(0)}(u)} & \frac{1}{Y^{(0)}(u)} + \frac{1}{Y^{(1)}(u)} \end{bmatrix}$$

Some results in our example situation

★ $\mathbb{X}(u)$:

$$\mathbb{X}(u) = \begin{bmatrix} Y_1(u) & Y_1(u)x_1 \\ Y_2(u) & Y_2(u)x_2 \\ \vdots & \vdots \\ Y_n(u) & Y_n(u)x_n \end{bmatrix}$$

★ $\mathbb{X}(u)^T \mathbb{X}(u)$:

$$\mathbb{X}(u)^T \mathbb{X}(u) = \begin{bmatrix} Y_{\bullet}(u) & Y^{(1)}(u) \\ Y^{(1)}(u) & Y^{(1)}(u) \end{bmatrix}$$

★ $(\mathbb{X}(u)^T \mathbb{X}(u))^{-1}$:

$$(\mathbb{X}(u)^T \mathbb{X}(u))^{-1} = \begin{bmatrix} \frac{1}{Y^{(0)}(u)} & -\frac{1}{Y^{(0)}(u)} \\ -\frac{1}{Y^{(0)}(u)} & \frac{1}{Y^{(0)}(u)} + \frac{1}{Y^{(1)}(u)} \end{bmatrix}$$

$$\mathbb{X}^-(u) = \begin{bmatrix} \frac{Y_1(u)(1-x_1)}{Y^{(0)}(u)} & \frac{Y_2(u)(1-x_2)}{Y^{(0)}(u)} & \dots & \frac{Y_n(u)(1-x_n)}{Y^{(0)}(u)} \\ \frac{Y_1(u)x_1}{Y^{(1)}(u)} - \frac{Y_1(u)(1-x_1)}{Y^{(0)}(u)} & \frac{Y_2(u)x_2}{Y^{(1)}(u)} - \frac{Y_2(u)(1-x_2)}{Y^{(0)}(u)} & \dots & \frac{Y_n(u)x_n}{Y^{(1)}(u)} - \frac{Y_n(u)(1-x_n)}{Y^{(0)}(u)} \end{bmatrix}$$

Some results in our situation

- ★ Resulting estimators

$$\widehat{B}_0(t) = \int_0^t \frac{J(u)}{Y^{(0)}(u)} dN^{(0)}(u) = \widehat{A}^0(t)$$

$$\begin{aligned}\widehat{B}_1(t) &= \int_0^t \frac{J(u)}{Y^{(1)}(u)} dN^{(1)}(u) - \int_0^t \frac{J(u)}{Y^{(0)}(u)} dN^{(0)}(u) \\ &= \widehat{A}^{(1)}(t) - \widehat{A}^{(0)}(t)\end{aligned}$$

- ★ From (4.64) in ABG:

$$\widehat{\Sigma}_{mod}(t) = \sum_{j: T_j \leq t} J(T_j) \mathbb{X}^-(T_j) \text{diag} \left\{ \mathbb{X}(T_j) \Delta \widehat{\mathbb{B}}(T_j) \right\} \mathbb{X}^-(T_j)^T$$