Plan for the exercise

- From $f(t)$, find $S(t)$ and $\alpha(t)$ for the exponential distribution
- Problem 1.3 in ABG: Show $E(T) = \int_0^\infty S(t)dt$
  - use this to find $E[T]$ in exponential and Weibull distribution
- Problem 1.9 in ABG: Truncated and censored survival times
  - survival function for left-truncated survival times
  - hazard rate for left-truncated survival times
  - intensity process for trucated and right-censored survival times

Relations between $f(t)$, $F(t)$, $\alpha(t)$, $S(t)$ and $A(t)$

$$
\begin{aligned}
f(t) &= F'(t) \\
F(t) &= \int_0^t f(s)ds \\
S(t) &= 1 - F(t) \\
f(t) &= -S'(t) \\
A(t) &= \int_0^t \alpha(s)ds \\
\alpha(t) &= A'(t) \\
\alpha(t) &= -\frac{d}{dt}\ln S(t) \\
S(t) &= \exp\{-A(t)\}
\end{aligned}
$$

Counting process and the intensity process

⋆ Counting process:

- consider particular type of event
- at most one event happens in short time interval $(t, t + dt]$
- $N(t)$: # of events from time 0 to time (including) time t
- increment: $dN(t) = N(t + dt) - N(t)$

⋆ Intensity process $\lambda(t)$:

$$\lambda(t)dt = P(dN(t) = 1|\text{past}) = E[dN(t)|\text{past}]$$

⋆ Corresponding martingale

$$M(t) = N(t) - \int_0^t \lambda(s)ds$$