Plan for this exercise session

- ⋆ For a simple parametric model with one parameter

  - – find likelihood and log likelihood functions
  - – find the maximum likelihood estimator
  - – find an estimator for the variance of the MLE
  - – find 95% confidence interval

- ⋆ For a parametric model with two parameters

  - – find likelihood and log likelihood functions
  - – derive formula for observed information matrix

Parametric counting process models

* ⋆ Situation

  - *n* individuals
  - individual *i* has hazard rate and intensity process

  $$\alpha(t|x_i) = \alpha_0(t; \theta) r(\beta, x_i)$$
  $$\lambda_i(t; \theta, \beta) = Y_i(t) \alpha_0(t; \theta) r(\beta, x_i)$$

  - aggregated intensity process

  $$\lambda_\bullet(t; \theta, \beta) = \sum_{i=1}^{n} \lambda_i(t, \theta, \beta)$$

* ⋆ Likelihood function

  $$L(\theta, \beta) = \left[\prod_{i=1}^{n} \prod_{0 < t \le \tau} \lambda_i(t; \theta, \beta)^{\Delta N_i(t)} \right] \exp\left\{ -\int_0^\tau \lambda_\bullet(t; \theta, \beta) dt \right\}$$

* ⋆ Log-likelihood function

  $$\ell(\theta, \beta) = \ln L(\theta, \beta)$$

Assymptotic properties

★ The maximum likelihood estimators are

$$\left(\widehat{\theta}, \widehat{\beta}\right) = \underset{(\theta, \beta)}{\mathsf{argmax}} \, \ell(\theta, \beta)$$

★ Score function

$$U(\theta, \beta) = \nabla \ell(\theta, \beta)$$

★ Information matrix

$$\mathbb{I}(\theta, \beta) = -\nabla^2 \ell(\theta, \beta)$$

★ We have approximately that

$$\left(\widehat{\theta}, \widehat{\beta}\right) \sim \mathsf{N}\left(\left[\begin{array}{c} \theta \\ \beta \end{array}\right], \mathbb{I}\left(\widehat{\theta}, \widehat{\beta}\right)^{-1}\right)$$

where θ and β are the true values.