Plan for this exercise session

- In a situation without truncation or censoring
  - simplify the expression for the Kaplan-Meier estimator
  - simplify the expression for the estimator for the variance

Situation

- We assume:
  - n individuals
  - each individual has the same $\alpha(t)$ and $S(t)$
  - may have truncation and/or censoring
  - $Y(t)$: number of individuals at risk just before time t

- Gives a multiplicative intensity process

$$\lambda(t) = \alpha(t)Y(t)$$

- Kaplan-Meier estimator

$$\widehat{S}(t) = \prod_{u<t}(1 - d\widehat{A}(u)) = \prod_{j:T_j \leq t}\left(1 - \frac{1}{Y(T_j)}\right)$$

Estimator for Var$[\widehat{S}(t)]$

- Using martingale theory, we found that
  - $\widehat{S}(t)$ is approximately normal
  - $\text{Var}[\widehat{S}(t)] = S^2(t)\text{Var}[\widehat{A}(t)]$

- Estimator for Var$[\widehat{S}(t)]$

$$\widehat{\tau}^2(t) = \widehat{S}^2(t)\widehat{\sigma}^2(t) = \widehat{S}^2(t) \sum_{j: T_j \le t} \frac{1}{Y(T_j)^2}$$

- Alternative estimator (Greenwood's formula)

$$\widetilde{\tau}^2(t) = \widehat{S}^2(t) \sum_{j: T_j \le t} \frac{1}{Y(T_j)(Y(T_j) - 1)}$$