Plan for this exercise session

- ⋆ Problem 4.3 in ABG:
  - counting processes: $N_i(t), i = 1, \ldots, n$
  - intensity process: $\lambda_i(t) = Y_i(t)\beta_0(t)$ where $Y_i(t) \in \{0, 1\}$
  - find expression for $\widehat{B}_0(t)$

- ⋆ Problem 4.8 in ABG:
  - additive regression model with $p = 2$ covariates

$$\alpha(t; x_1, x_2) = \beta_0(t) + \beta_1(t)x_1 + \beta_2(t)x_2$$

  - assume x_1 and x_2 are realisations of stochastic variables
  - what if one of the covariates is dropped from the model?
  - first: x_1 and x_2 are independent
  - thereafter: (x_1, x_2) are bivariate Gaussian

Additive regression models — situation

- n individuals
- individual i has covariate vector $x_i(t)$
- individual i has hazard rate and intensity process

$$\alpha(t|x_i(t)) = \alpha_0(t)r(\beta, x_i(t))$$
$$\lambda_i(t) = Y_i(t)\alpha_0(t)r(\beta, x_i(t))$$

  where

$$r(\beta, x_i(t)) = \beta_0(t) + \beta_1(t)x_{i1}(t) + \ldots \beta_p(t)x_{ip}(t)$$

- $N_i(t)$, $Y_i(t)$, $N_\bullet(t)$, $Y_\bullet(t)$, $\lambda_\bullet(t)$

⋆ $B_q(t) = \int_0^t \beta_q(u)du$ for $q = 0, 1, \ldots, p$

Additive regression models — notation and estimator

- $\mathbb{N}(u) = [N_1(u) \;\; N_2(u) \;\; \cdots \;\; N_n(u)]^T$
- $\mathbb{X}(u)$:

$$\mathbb{X}(u) = \begin{bmatrix} Y_1(u) & Y_1(u)x_{11}(u) & \cdots & Y_1(u)x_{1p}(u) \\ Y_2(u) & Y_2(u)x_{21}(u) & \cdots & Y_2(u)x_{2p}(u) \\ \vdots & \vdots & & \vdots \\ Y_n(u) & Y_n(u)x_{n1}(u) & \cdots & Y_n(u)x_{np}(u) \end{bmatrix}$$

- $\mathbb{X}^-(u) = (\mathbb{X}(u)^T\mathbb{X}(u))^{-1}\mathbb{X}(u)^T$, $(p+1) \times n$ matrix
- $J(u) = \mathbb{I}(\mathbb{X}(u)$ has full rank), scalar
- Estimator

$$\widehat{\mathbb{B}}(t) = \begin{bmatrix} \widehat{B}_0(t) \\ \widehat{B}_1(t) \\ \vdots \\ \widehat{B}_p(t) \end{bmatrix} = \int_0^t J(u)\mathbb{X}^-(u)d\mathbb{N}(u)$$

Bivariate Gaussian and condition distribution

- ⋆ Let

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \sim N\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right)$$

- ⋆ Then we have

$$x_2 | x_1 \sim N\left(\mu_2 + \frac{\rho\sigma_2}{\sigma_1}(x_1 - \mu_1), (1-\rho^2)\sigma_2^2 \right)$$

- ⋆ Common notation

$$\mu_{2|1} = E[x_2|x_1] = \mu_2 + \frac{\rho\sigma_2}{\sigma_1}(x_1 - \mu_1)$$

$$\sigma_{2|1}^2 = \mathrm{Var}[x_2|x_1] = (1-\rho^2)\sigma_2^2$$