Plan for the lecture

- ⋆ The Weibull distribution
  - hazard rate, survival function, cumulative distribution function, density function
- ⋆ The laws of total probability and double expectation
  - discrete stochastic variables
  - continuous stochastic variables
  - conditioning on histories
- ⋆ A counting process example

Relations between $f(t)$, $F(t)$, $\alpha(t)$, $S(t)$ and $A(t)$

$$
\begin{aligned}
f(t) &= F'(t) \\
F(t) &= \int_0^t f(s)ds \\
S(t) &= 1 - F(t) \\
f(t) &= -S'(t) \\
A(t) &= \int_0^t \alpha(s)ds \\
\alpha(t) &= A'(t) \\
\alpha(t) &= -\frac{d}{dt} \ln S(t) \\
S(t) &= \exp\{-A(t)\}
\end{aligned}
$$

Counting process and the intensity process

- Counting process:
  - consider particular type of event
  - at most one event happens in short time interval $(t, t + dt]$
  - $N(t)$: # of events from time 0 to time (including) time t
  - increment: $dN(t) = N(t + dt) - N(t)$
- Intensity process $\lambda(t)$:

$$\lambda(t)dt = P(dN(t) = 1|\text{past}) = E[dN(t)|\text{past}]$$

- Corresponding martingale

$$M(t) = N(t) - \int_0^t \lambda(s)ds$$