

Plan for this lecture

- ★ Recall some theory parts
 - parametric counting process models — situation
 - likelihood function
 - maximum likelihood estimators
 - asymptotic properties
- ★ Exponential regression
 - with one (categorical) covariate
- ★ Weibull regression
 - with one (categorical) covariate

Parametric counting process models

★ Situation

- n individuals
- individual i has hazard rate and intensity process

$$\alpha(t|x_i) = \alpha_0(t; \theta)r(\beta, x_i)$$

$$\lambda_i(t; \theta, \beta) = Y_i(t)\alpha_0(t; \theta)r(\beta, x_i)$$

- aggregated intensity process

$$\lambda_{\bullet}(t; \theta, \beta) = \sum_{i=1}^n \lambda_i(t, \theta, \beta)$$

★ Likelihood function

$$L(\theta, \beta) = \left[\prod_{i=1}^n \prod_{0 < t \leq \tau} \lambda_i(t; \theta, \beta)^{\Delta N_i(t)} \right] \exp \left\{ - \int_0^{\tau} \lambda_{\bullet}(t; \theta, \beta) dt \right\}$$

★ Log-likelihood function

$$\ell(\theta, \beta) = \ln L(\theta, \beta)$$

Asymptotic properties

- ★ The maximum likelihood estimators are

$$\left(\widehat{\theta}, \widehat{\beta}\right) = \operatorname{argmax}_{(\theta, \beta)} \ell(\theta, \beta)$$

- ★ Score function

$$U(\theta, \beta) = \nabla \ell(\theta, \beta)$$

- ★ Information matrix

$$\mathbb{I}(\theta, \beta) = -\nabla^2 \ell(\theta, \beta)$$

- ★ We have approximately that

$$\left(\widehat{\theta}, \widehat{\beta}\right) \sim \mathbf{N}\left(\begin{bmatrix} \theta \\ \beta \end{bmatrix}, \mathbb{I}\left(\widehat{\theta}, \widehat{\beta}\right)^{-1}\right)$$

where θ and β are the true values.