Plan for this lecture

- ⋆ Brief summary of Kaplan-Meier estimator
  - Kaplan-Meier estimator, $\widehat{S}(t)$
    - product-integral
  - estimator for variance of $\widehat{S}(t)$
  - large sample properties of $\widehat{S}(t)$
  - estimation of median survival times
- ⋆ Confidence interval for $S(t)$
  - includes solving Problem 3.6 in ABG
- ⋆ In a simple linear regression model: How to find a confidence interval for x for a given value of μ
  - related to confidence interval for the pth fractile from $\widehat{S}(t)$
- ⋆ Discuss examples in ABG
  - example 3.8 (Figures 3.11 and 3.13)
  - example 3.9 (Figure 3.12)

Situation

- We assume:
  - n individuals
  - each individual has the same $\alpha(t)$ and $S(t)$
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⋆ Kaplan-Meier estimator

$$\widehat{S}(t) = \prod_{u<t}(1 - d\widehat{A}(u)) = \prod_{j:T_j \leq t}\left(1 - \frac{1}{Y(T_j)}\right)$$

Estimator for $\text{Var}[\widehat{S}(t)]$

- Using martingale theory, we found that
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- Alternative estimator (Greenwood's formula)

$$\widehat{\tau}^2(t) = \widehat{S}^2(t) \sum_{j:T_j \le t} \frac{1}{Y(T_j)(Y(T_j)-1)}$$

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- Obtain confidence interval by including in the interval all values ξ_p^0 that rejects $H_0 : \xi_p = \xi_p^0$ when tested against $H_1 : \xi_p \neq \xi_p^0$