Plan for this lecture

- Summary of nonparametric tests for two groups of individuals
  - test for two groups, $H_0 : \alpha_1(t) = \alpha_2(t)$
- Nonparametric test with only one group av individuals

$$H_0 : \alpha(t) = \alpha_0(t)$$

Nonparametric test for two groups

- Two counting processes: $N_1(t)$ and $N_2(t)$

$$\lambda_1(t) = \alpha_1(t) Y_1(t) \quad \text{and} \quad \lambda_2(t) = \alpha_2(t) Y_2(t)$$

- Want to test $H_0 : \alpha_1(t) = \alpha_2(t)$ for $t \in [0, t_0]$
- Consider statistic

$$\begin{aligned} Z_1(t_0) &= \int_0^{t_0} L(t)(d\widehat{A}_1(t) - d\widehat{A}_2(t)) \\ &= \int_0^{t_0} \frac{L(t)}{Y_1(t)} dN_1(t) - \int_0^{t_0} \frac{L(t)}{Y_2(t)} dN_2(t) \end{aligned}$$

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- Use Doob-Meyer decompositions of $N_1(t)$ and $N_2(t)$ and get

$$Z_1(t_0) = \int_0^{t_0} L(t)(\alpha_1(t) - \alpha_2(t))dt + \int_0^{t_0} \frac{L(t)}{Y_1(t)} dM_1(t) - \int_0^{t_0} \frac{L(t)}{Y_2(t)} dM_2(t)$$

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- ⋆ When H_0 is true $Z_1(t_0)$ is a mean zero martingale

Properties of $Z_1(t_0)$ when H_0 is true

- $\mathrm{E}[Z_1(t_0)] = 0$
- Using the predictable variation process $\langle Z_1 \rangle(t_0)$ we found

$$\mathrm{Var}[Z_1(t_0)] = \mathrm{E}\left[\int_0^{t_0} \frac{L^2(t)(Y_1(t)+Y_2(t))}{Y_1(t)Y_2(t)}\alpha(t)dt\right]$$

- Estimating $\alpha(t)dt$ with $d\widehat{A}(t)$ (data from both groups) we get an estimator for $\mathrm{Var}[Z_1(t_0)]$

$$V_{11}(t_0) = \int_0^{t_0} \frac{L^2(t)}{Y_1(t)Y_2(t)} dN_\bullet(t)$$

- It can be shown (Chapter 3.3.5 in ABG) that $Z_1(t_0)$ is approximately normal
- Use test statistic

$$U(t_0) = \frac{Z_1(t_0)}{\sqrt{V_{11}(t_0)}}$$

  which is approximately standard normal when H_0 is true

One sample nonparametric test

- Assume:
  - n individuals
  - each individual has the same hazard rate $\alpha(t)$
  - no tied observations
  - $N(t)$: # individuals failed up to (and including) time t
  - $Y(t)$: # individuals at risk just before time t
- Multiplicative intensity model

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where $\alpha_0(t)$ is a known hazard rate

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- Want to define the test statistic to use!