Plan for this lecture

- ⋆ Recall some theory parts
  - partial likelihood function for a relative risk model
  - score test for a relative risk model
  - two-sample non-parametric log-rank test
- ⋆ Cox regression with one binary covariate
- ⋆ Score test for a Cox regression situation with one binary covariate — two-sample log-rank test

Partial likelihood function for a relative risk model

- Situation:
  - n individuals
  - individual i has covariate vector $x_i(t)$
  - individual i has hazard rate and intensity process

$$\begin{aligned}\alpha(t|x_i(t)) &= \alpha_0(t)r(\beta, x_i(t)) \\ \lambda_i(t) &= Y_i(t)\alpha_0(t)r(\beta, x_i(t))\end{aligned}$$

  - $N_i(t)$, $Y_i(t)$, $N_\bullet(t)$, $Y_\bullet(t)$, $\lambda_\bullet(t)$

- Partial likelihood

$$L(\beta) = \prod_j \left[\frac{r(\beta, x_{i_j}(T_j))}{\sum_{\ell=1}^n Y_\ell(T_j)r(\beta, x_\ell(T_j))}\right]$$

- For Cox regression models:

$$r(\beta, x_i(t)) = \exp\{\beta^T x_i(t)\}$$

Score test for a relative risk model

- Situation:
  - n individuals
  - individual i has covariate vector $x_i(t)$
  - individual i has hazard rate and intensity process

$$\begin{aligned} \alpha(t|x_i(t)) &= \alpha_0(t) r(\beta, x_i(t)) \\ \lambda_i(t) &= Y_i(t)\alpha_0(t) r(\beta, x_i(t)) \end{aligned}$$

  - $N_i(t)$, $Y_i(t)$, $N_\bullet(t)$, $Y_\bullet(t)$, $\lambda_\bullet(t)$
- Score test: $H_0 : \beta = \beta_0$
  - test statistic
$$\chi^2_{SC} = U(\beta_0)^T \mathbb{I}(\beta_0)^{-1} U(\beta_0)$$
    where χ^2_{SC} is approximately χ^2-distributed with q degrees of freedom when H_0 is true
  - $U(\beta) = \nabla \ell(\beta)$
  - $\mathbb{I}(\beta) = -\nabla U(\beta) = -\nabla^2 \ell(\beta)$

Two-sample nonparametric log-rank test

- Have two counting processes $N_1(t)$ and $N_2(t)$ with intensity processes

$$\lambda_1(t) = Y_1(t)\alpha_1(t)$$
$$\lambda_2(t) = Y_2(t)\alpha_2(t)$$

- Log-rank test: $H_0 : \alpha_1(t) = \alpha_2(t), t \in [0, t_0]$

– test statistic

$$\chi^2_{LR} = \frac{Z_1(t_0)^2}{V_{11}(t_0)}$$

where χ^2_{LR} is approximately χ^2 distributed with 1 degree of freedom when H_0 is true

– $Z_1(t_0) = \int_0^{t_0} \frac{L(t)}{Y_1(t)} dN_1(t) - \int_0^{t_0} \frac{L(t)}{Y_2(t)} dN_2(t)$

– $V_{11}(t_0) = \int_0^{t_0} \frac{L^2(t)}{Y_1(t)Y_2(t)} dN_\bullet(t)$

– $L(t) = \frac{Y_1(t)Y_2(t)}{Y_\bullet(t)}$