Plan for this lecture

- ⋆ Briefly recall some theory parts
  - additive regression models
  - situation
  - notation
  - estimators
- ⋆ Additive regression model with one binary covariate
  - derive formulas for the estimators
  - relation to the Nelson-Aalen estimator

Additive regression models — situation

- n individuals
- individual i has covariate vector $x_i(t)$
- individual i has hazard rate and intensity process

$$\alpha(t|x_i(t)) = \alpha_0(t) r(\beta, x_i(t))$$
$$\lambda_i(t) = Y_i(t)\alpha_0(t) r(\beta, x_i(t))$$

where

$$r(\beta, x_i(t)) = \beta_0(t) + \beta_1(t)x_{i1}(t) + \ldots \beta_p(t)x_{ip}(t)$$

- $N_i(t)$, $Y_i(t)$, $N_\bullet(t)$, $Y_\bullet(t)$, $\lambda_\bullet(t)$

⋆ $B_q(t) = \int_0^t \beta_q(u)du$ for $q = 0, 1, \ldots, p$

Additive regression models — notation and estimator

- $\mathbb{N}(u) = [N_1(u)N_2(u)\cdots N_n(u)]^T$
- $\mathbb{X}(u)$:

$$\mathbb{X}(u) = \begin{bmatrix} Y_1(u) & Y_1(u)x_{11}(u) & \cdots & Y_1(u)x_{1p}(u) \\ Y_2(u) & Y_2(u)x_{21}(u) & \cdots & Y_2(u)x_{2p}(u) \\ \vdots & \vdots & & \vdots \\ Y_n(u) & Y_n(u)x_{n1}(u) & \cdots & Y_n(u)x_{np}(u) \end{bmatrix}$$

- $\mathbb{X}^-(u) = (\mathbb{X}(u)^T\mathbb{X}(u))^{-1}\mathbb{X}(u)^T$, $(p+1) \times n$ matrix
- $J(u) = \mathbb{I}(\mathbb{X}(u)$ has full rank), scalar
- Estimator

$$\widehat{\mathbb{B}}(t) = \begin{bmatrix} \widehat{B}_0(t) \\ \widehat{B}_1(t) \\ \vdots \\ \widehat{B}_p(t) \end{bmatrix} = \int_0^t J(u)\mathbb{X}^-(u)d\mathbb{N}(u)$$