Eksamen 5075 Levetidsanalyse 10.5.2000

Løsningsforslag

1

Oppgave 1

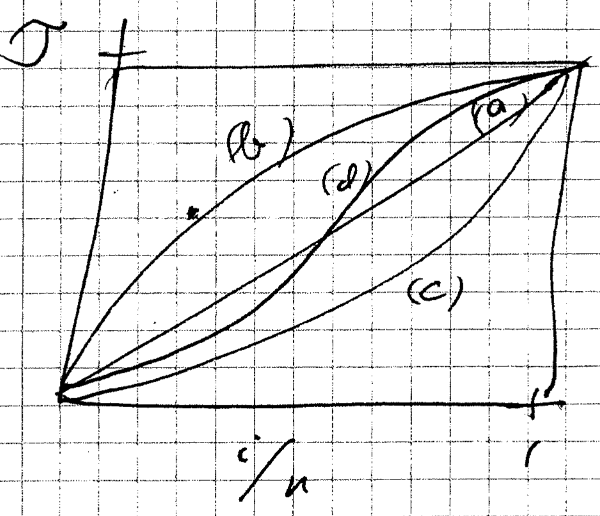
TTT ved tid $T_{(k)}$ $= \mathcal{T}(T_{(k)}) = \sum_{i=1}^{k-1} T_{(i)} + (n-k+1)\,T_{(k)}$

Total "operasjonstid" for alle komp. opp til tid T_k

$$\mathcal{T}(T_{(n)}) = \sum_{i=1}^{n-1} T_{(i)} + T_{(n)} = \sum_{i=1}^{n} T_i$$

TTT-plott:

$$\left(\frac{i}{n}, \frac{\mathcal{T}(T_{(i)})}{\mathcal{T}(T_{(n)})}\right) \quad i=1,\dots,n$$



(a) Konstant sviktint

(b) voksende sviktint

(c) avtakende sviktint

(d) Badekars form

(H & R s. 356 – 368)

Oppgave 2

(a) T_i = tid til svikt

S_i = tid til sensur. $\qquad i = 1, \dots, n$

(H & R s. 396)

observer: $Y_i = \min(T_i, S_i)$

$$\delta_i = \begin{cases} 1 & T_i \le S_i \\ 0 & T_i > S_i \end{cases}$$

$$l(\lambda) = \prod_{i=1}^{n} \left[f(Y_i;\lambda)^{\delta_i} \, R(Y_i;\lambda)^{1-\delta_i} \right]$$

$$= \prod_{i=1}^{n} \left[\lambda e^{-\lambda Y_i}\right]^{\delta_i} \left[e^{-\lambda Y_i}\right]^{1-\delta_i}$$

$$= \lambda^{\sum_1^n \delta_i} e^{-\lambda \cdot \sum Y_i}$$

La $r = \sum_{i=1}^{n} \delta_i$ = antall observerte svikt

$$\underline{l(\lambda) = \lambda^r e^{-\lambda \sum_1^n Y_i}}$$

(b) $\ln(l(\lambda)) = r \ln \lambda - \lambda \sum_1^n Y_i$

$$\frac{\partial \ln l(\lambda)}{\partial \lambda} = \frac{r}{\lambda} - \sum Y_i \implies \underline{\lambda^* = \frac{r}{\sum_1^n Y_i}}$$

$$\frac{\partial^2 \ln l(\lambda)}{\partial \lambda^2} = -\frac{r}{\lambda^2}$$

Asymptotisk er $Var(\lambda^*) \approx \dfrac{\lambda^2}{r}$

Asymptot. fordel. for λ^*

$$\sim N\left(\lambda, \frac{\lambda^2}{r}\right)$$

$$\underline{\lambda^*} = \frac{5}{\sum Y_i} = \frac{5}{2370} = \underline{0.00211}$$

(3)

c) (i) $$\frac{\lambda^* - \lambda}{\lambda}\sqrt{r} \sim N(0,1)$$

$$P\left(-u_{\frac{\alpha}{2}} < \left(\frac{\lambda^*}{\lambda} - 1\right)\sqrt{r} < u_{\frac{\alpha}{2}}\right) \approx 1-\alpha$$

$$P\left[\frac{\lambda^*}{1+\frac{u_{\alpha/2}}{\sqrt{r}}} < \lambda < \frac{\lambda^*}{1-\frac{u_{\alpha/2}}{\sqrt{r}}}\right] \approx 1-\alpha$$

(ii) $$2\lambda\sum_{i=1}^{n} Y_i \sim \chi^2_{2r}\text{ - fordelt}$$

$$P\left[z_{2r;1-\alpha/2} < 2\lambda\sum_{1}^{n} Y_i < z_{2r;\alpha/2}\right] = 1-\alpha$$

$$P\left[\frac{z_{2r;1-\alpha/2}}{2\sum Y_i} < \lambda < \frac{z_{2r;\alpha/2}}{2\sum Y_i}\right] = 1-\alpha$$

((iii) $$2\left[\ln \ell(\lambda^*) - \ln \ell(\lambda)\right] \sim \chi^2_1$$

s. 58–59

$$P\left[2(\ln \ell(\lambda^*) - \ln \ell(\lambda)) < z_{1,\alpha}\right] = 1-\alpha$$

$$P\left[\ln(\ell(\lambda)) > \ln(\ell(\lambda^*)) - \tfrac{1}{2} z_{1;\alpha}\right] = 1-\alpha$$)

Tallregning: $\alpha = 0.10$

(i) $$\left(\frac{0.0021}{1+\frac{1.645}{\sqrt{5}}}, \frac{0.0021}{1-\frac{1.645}{\sqrt{5}}}\right) \qquad (0.0012, 0.0079)$$

(ii) $$\left(\frac{3.94}{2\cdot 2370}, \frac{18.31}{2\cdot 2370}\right) \qquad (0.0008, 0.0039)$$

d) $P(T > 600) = R(600) = e^{-\lambda 600}$

SME for denne sannsh er

$$R^* = e^{-\lambda^* 600} = \underline{\underline{0.282}}$$

(~~Stor~~ ut fra ~~dataene~~ ?)

e) Weibull: $\underline{R(t) = e^{-(\lambda t)^\alpha}}$

$\underline{H_0: \alpha = 1}$ $\quad$ $\underline{H_1: \alpha \neq 1}$

Forkast H_0 når

$$W = 2\left[\ln l(\alpha^*, \lambda^*) - \ln l(1, \hat{\lambda})\right] > z_{1,0.05}$$

(Her er $\hat{\lambda}$ SME funnet i b)

Fra (b) har vi

$$\ln(l(1,\hat{\lambda})) = r \ln \hat{\lambda} - \hat{\lambda} \sum_{i}^{6} Y_i$$

$$= r \ln \hat{\lambda} - \cancel{r} = r(\ln(\hat{\lambda}) - 1)$$

$$= \underline{-35.81}$$

~~Det~~ er oppgitt at

$$\ln l(\alpha^*, \lambda^*) = \underline{-33.11}$$

$$\underline{W} = 2[-33.11 + 35.81] = 2 \cdot 2.7 = \underline{5.4} > \underline{3.84}$$

Forkast H_0. Vi kan påstå at vi ikke har ekspon. f

(jfr. d): I ny modell blir $R^* = e^{-(0.00215 \cdot 600)^{2.96}} = \underline{\underline{0.12}}$

Som virker rimeligere)

Oppgave 3

a (i) $R(t \mid \underline{z}) = R(\phi(\underline{z}) \cdot t \mid \underline{z} = 0) = R_0(\phi(\underline{z})t)$ (Ansel & Phil s. 66)

$\underline{\phi(0) = 1, \ \phi(\underline{z}) > 0}$

(ii) $R(t, s) = R_0(\lambda(s) \cdot t)$ H & R s. 42, s. 423

der

$\lambda(s) = c \cdot s^{a}$ $\quad cs^{-a}$

(s skalar) $\quad c\,e^{-b/s}$ Arrhenius

$\quad cs\,e^{-b/s}$ Eyring.

$u = \phi(z) \cdot t$

$$\underline{E(T \mid \underline{z})} = \int_0^\infty R(t \mid \underline{z})\,dt$$

$$\int_0^\infty R_0(\phi(\underline{z}) \cdot t)\,dt = \int_0^\infty R_0(u)\,\frac{du}{\phi(\underline{z})} = \underline{\frac{E(T \mid \underline{z} = 0)}{\phi(\underline{z})}}$$

(b) Svikt intensitet antas

$\underline{z(t; \underline{s}) = z_0(t) \cdot g(\underline{s})}$ (P–H)

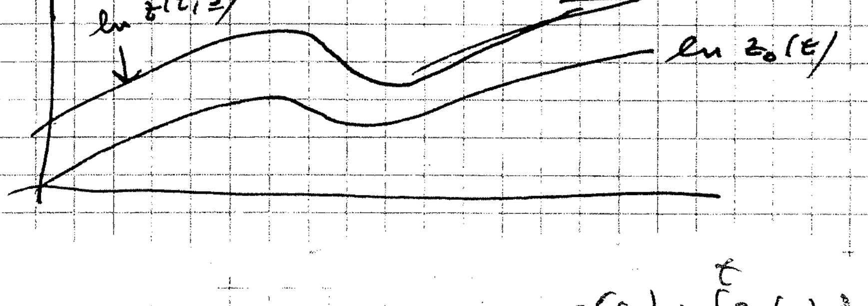
der $\underline{s}$ = stressor (kovariates)

$\underline{\text{Cox}}$: $g(\underline{s}) = e^{\sum_i \beta_i \cdot s_i} = e^{\underline{\beta}' \underline{s}}$

Sentral forutsetning: (proporsjonalitet) (H & R s 432–33)

Antar effekt av $\underline{s}$ uavhengig av tid

$\underline{\ln z(t, s) = \ln z_0(t) + \ln g(\underline{s})}$

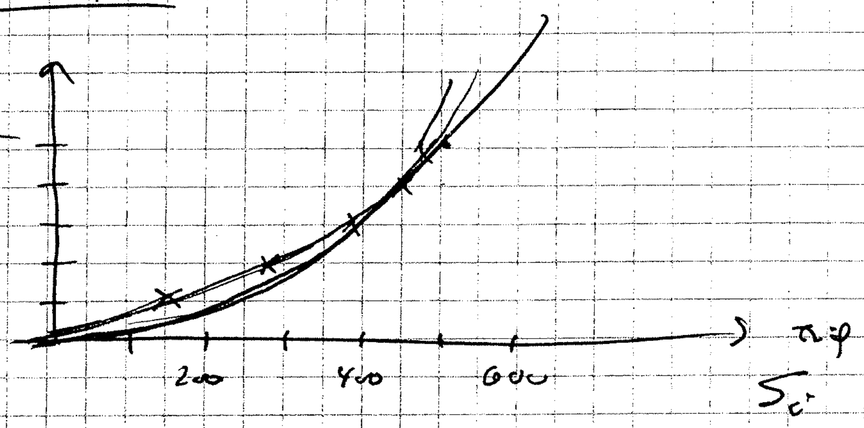


$$R(t; \underline{s}) = e^{-g(\underline{s}) \cdot \int_0^t z_0(u)\,du} = \left[e^{-\int_0^t z_0(u)\,du}\right]^{g(\underline{s})}$$

$$R(t; \underline{s}) = R(t; \underline{s}^{(0)})^{g(\underline{s})}$$

Oppgave 4

a



H&R s. 321:

Mil. Hdbk test

$$Z = 2\sum_{1}^{n} \ln \frac{t_0}{S_i} \qquad (t_0 = \tau = 500)$$

Forkast H_0: ingen trend, hvis $Z > z_{\frac{\alpha}{2}; 2n}$

eller $Z < z_{1-\alpha/2; 2n}$

$$\underline{Z} = 2n\left[\ln(t_0)\right] - 2\sum_{1}^{n} \ln S_i$$

$$= 10 \cdot \ln(500) - 2 \cdot 28.93 = \underline{\underline{4.29}}$$

$$(62.15 - 57.86)$$

$$z_{0.95;10} = \underline{3.94} \qquad z_{0.05;10} = \underline{18.31}$$

Ikke forkasting (selv ved nivå 10%)

b

$w(t) = \lambda\beta \cdot t^{\beta-1}$, "Power Law" H&R s. 3

$$\underline{W(t)} = E(N(t)) = \lambda \cdot t^{\beta} = \int_0^t w(u)\,du$$

$$l(\lambda,\beta) = \prod_{i=1}^{5}\left(\lambda\beta \cdot S_i^{\beta-1}\right) \cdot e^{-\lambda\tau^{\beta}}$$

$$\ln l(\lambda,\beta) = 5\ln(\lambda) + 5\ln(\beta) + (\beta-1)\ln(S_1 \cdots S_5) - \lambda\tau^{\beta}$$

c) $$\frac{\partial \ln \ell(\lambda,\beta)}{\partial \lambda} = \frac{5}{\lambda} - \tau^{\beta}$$

$$\lambda^* = \frac{5}{\tau^{\beta^*}} = \frac{N(\tau)}{\tau^{\beta^*}}$$

$$\frac{\partial \ln \ell(\lambda,\beta)}{\partial \beta} = \frac{5}{\beta} + \sum_1^5 \ln(s_i) - \lambda \cdot \tau^{\beta} \cdot \ln(\tau)$$

$$\frac{5}{\beta^*} + \sum_1^5 \ln(s_i) - \underbrace{\lambda^* \tau^{\beta^*}}_{=5} \cdot \ln(\tau) = 0$$

$$\beta^* = \frac{5}{5 \cdot \ln(\tau) - \sum_1^5 \ln(s_i)} = \frac{N(\tau)}{N(\tau) \cdot \ln(\tau) - \sum_1^{N(\tau)} \ln(s_i)}$$

d) $$\beta^* = \frac{5}{5 \cdot \ln(500) - 28{,}93} = \frac{10}{Z} \quad (\text{se a)})$$

$$= \frac{10}{4.29} = 2.33$$

$$\lambda^* = \frac{5}{500^{2.33}} = 2.57 \cdot 10^{-6}$$

$W(200) = 0.59$ $\quad W(400) = 2.97$

$W(500) = 5.00$

e) H_0 $\beta=1$ (ingen trend) $\quad H_1: \beta \neq 1$

Förkast H_0 när $W = 2\left[\ln l(\lambda^*, \beta^*) - \ln l(\hat{\lambda}, 1)\right] > z_{\alpha,1}$

$$\ln l(\lambda^*, \beta^*) = 5\ln(\lambda^*) + 5\ln(\beta^*) + (\beta^*-1)\sum_1^5 \ln S_i - \underbrace{\lambda^* \tau^{\beta^*}}_{=5}$$

$$5 \cdot \ln\left[2.57 \cdot 10^{-6}\right] + 5\ln(2.33) + 1.33 \cdot 28.93 - 5$$

$$-64.36 + 4.23 + 38.48 - 5 = \underline{\underline{-26.65}}$$

När $\beta=1$ fås $\hat{\lambda} = \frac{5}{\tau} = \frac{1}{100}$

$$\ln l(\hat{\lambda}, 1) = 5\ln(\hat{\lambda}) + 0 + 0 - \underbrace{\hat{\lambda}\tau}_{=5}$$

$$= 5\left[-\ln 100 - 1\right] = \underline{\underline{-28.03}}$$

$$W = 2\left[-26.65 + 28.03\right] = \underline{\underline{2.76}}$$

$\underline{z_{0.05,1} = 3.84}$ ej förkast $\alpha = 5\%$

$\underline{z_{0.10,1}} = \underline{2.71}$ såvitt förkasting 10%

a) beta fordelingen er konjugert a priori ford. (Gir aposteriori ford i samme klasse)

likelihood: $l(p) \sim p^X (1-p)^{n-X}$

a priori: $f_P(p) \sim p^{r-1}(1-p)^{s-1}$

aposteriori: $f_{P|X}(p|x) \sim l(p)\cdot f_P(p) = p^{X+r-1}(1-p)^{n-X+s-1}$

a posteriori $\sim$ beta$(X+r,\ n-X+s)$

$$E(P|X) = \frac{X+r}{n+r+s} = \text{Bayes estimator}$$

b) A priori $E(p) = \frac{r}{r+s} = 0.02$

r tilsvarer analogt som X

r+s — " — n

$\Longrightarrow$ Velg (1) $r+s=10$ (2) $\frac{r}{10} = 0.02$

Dvs $r = 0.2$ $s = 9.8$

Bayes est. $\hat{p} = \frac{5+0.2}{90+0.2+9.8} = \frac{5.2}{100}$

$\hat{p} = 0.052$

Ikke-informativ på feks. $f_P(p) = 1$
(dvs $r = s = 1$)

$\hat{p} = \frac{5+1}{90+2} = \frac{6}{92} = 0.065$ ($>$ 0.056 "hlgs.sannsyn.")

(Evt Jeffrey's: $r = s = \frac{1}{2}$ $\hat{p} = \frac{5+\frac{1}{2}}{90+1} = 0.06$