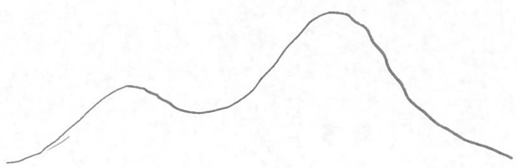
26.01.2016

Review:

- Method based on mixtures



- Multivariate normal distribution
- Rejection sampling algorithm

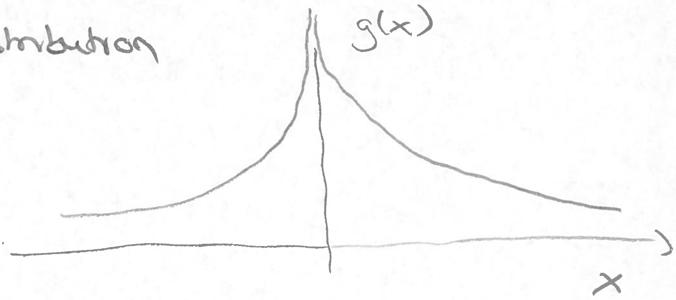
$$P\left(U \leq \frac{1}{c}\frac{f(\bar{X})}{g(\bar{X})}\right) = P(c \cdot g(\bar{X}) \cdot U \leq f(\bar{X})) = \frac{1}{c}$$

Example:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$g(x) = \frac{\lambda}{2} e^{-\lambda|x|} \qquad -\infty < x < \infty, \ \lambda > 0$$

$\uparrow$
double exponential distribution



<u>Note</u>: Can sample from $g(x)$ by

- generate $z \sim Exp(\lambda)$
- generate $y \in \{0, 1\}$ with $P(Y=1) = P(Y=0) = \frac{1}{2}$
- if $y = 0$

    set $x = -z$

  else

    set $x = z$

  end
  return x

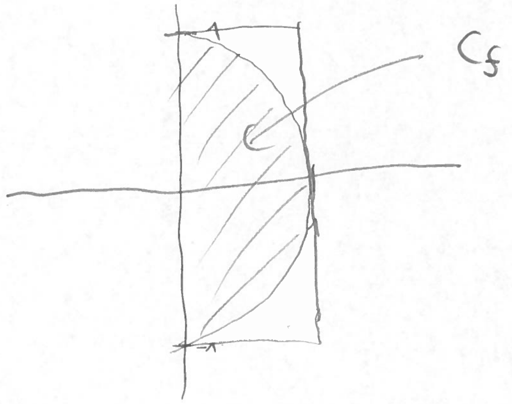
$$f(x) \leq c \cdot g(x) \iff \frac{f(x)}{g(x)} \leq c$$

$$c = \sqrt{\frac{2}{\pi}} \frac{1}{\lambda} e^{\frac{1}{2}\lambda^2}$$

$$\frac{dc}{d\lambda} = \sqrt{\frac{2}{\pi}} \left[-\frac{1}{\lambda^2} e^{\frac{1}{2}\lambda^2} + \frac{1}{\lambda} e^{\frac{1}{2}\lambda^2} \cdot \lambda \right]$$

$$= \sqrt{\frac{2}{\pi}} e^{\frac{1}{2}\lambda^2} \left[-\frac{1}{\lambda^2} + 1 \right] \stackrel{!}{=} 0 \Rightarrow \lambda = 1$$

Standard-Cauchy
example



$C_f = \{(x_1, x_2) \mid x_1^2 + x_2^2 \leq 1,\ x_1 > 0\}$

Let $g(x_1,x_2) = \begin{cases} \frac{1}{2}, & x_1 \in [0,1],\ x_2 \in [-1,1] \\ 0, & \text{otherwise} \end{cases}$

Thus the density g is that $x_1 \sim U(0,1)$ independently
$x_2 \sim U(-1,1)$

The rejection algorithm becomes:

1). Generate $(x_1, x_2) \sim g(x_1, x_2)$

2.) Compute

$\frac{1}{|C_f|}$ $(x_1,x_2) \in C_f$ $c = \frac{2}{|C_f|}$

$$\alpha = \frac{1}{c} \frac{f(x_1,x_2)}{g(x_1,x_2)} = \begin{cases} \frac{1}{c} \cdot \frac{2}{|C_f|} = 1 & \text{if } (x_1,x_2) \in C_f \\ 0 & \text{else} \end{cases}$$

3.) [$u \sim U(0,1)$
If $(u \leq \alpha)$ you are finished
return $\frac{x_2}{x_1}$]

If $(x_1, x_2) \in C_f$
finished (check $x_1^2 + x_2^2 \leq 1$)

4.) otherwise go back to 1