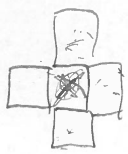
01.03.2016



2 Stage

$(\underline{\underline{x_i}} \sim \underline{N(0, \sigma^2)})$

3. stage

$\underline{y_i}$

E_i

$$y_i \sim N(\mu_i, \sigma^2)$$

Identity

$$\mu_i = \alpha + \beta_1 x_i + v_i$$

$$y_i | \eta_i \sim Po(E_i \underline{\underline{\lambda_i}})$$

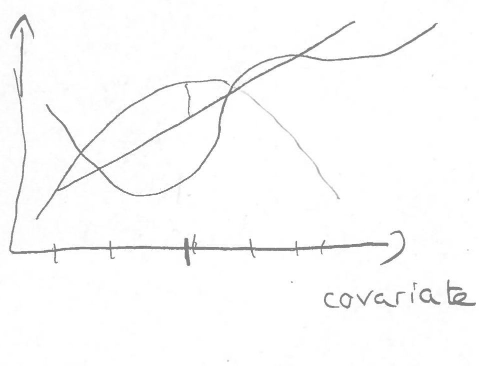
$$\eta_i = \log(\lambda_i) = \alpha + \beta_1 x_i + v_i$$

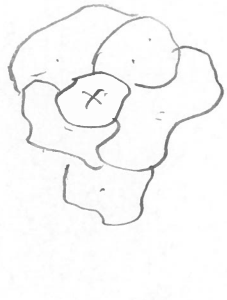
$$y_i | \pi_i \sim Bin(n_i, \underline{\underline{\pi_i}})$$

$$\text{logit}(\pi_i) = \alpha + \beta_1 x_i + v_i$$

c_i

$$\log\left(\frac{\pi_i}{1-\pi_i}\right)$$





Definition: GMRF

A random vector $x = (x_1, \dots, x_n)^T$ is called GMRF with respect to a labelled graph $G = (V, \mathcal{E})$ with mean vector μ and precision matrix (inverse covariance matrix) $Q > 0$, if its density has the form

$$\pi(x) = (2\pi)^{-n/2} \, |Q|^{1/2} \exp\left(-\frac{1}{2}(x-\mu)^T Q (x-\mu)\right)$$

and $Q_{ij} \neq 0 \iff \{i, j\} \in \mathcal{E}$ for all $i \neq j$.

$\Rightarrow$ any normal distribution with a symmetric positive definite covariance matrix is also a GMRF and vize versa.

$$\pi(x \mid \theta) = \frac{\pi(x,\theta)}{\pi(\theta)}$$

$$\pi(x \mid \theta, y) = \frac{\pi(x,\theta \mid y)}{\pi(\theta \mid y)}$$

$$\Rightarrow \pi(\theta \mid y) = \frac{\pi(x,\theta \mid y)}{\pi(x \mid \theta, y)} = \frac{\pi(x,\theta,y)}{\underbrace{\pi(y)}_{\text{unknown}}\,\pi(x \mid \theta, y)}$$

$$\propto \frac{\pi(x,\theta,y)}{\pi(x \mid \theta, y)} = \frac{\pi(\theta)\,\pi(x,y \mid \theta)}{\pi(x \mid \theta, y)}$$