

expected cost function

$$ECM(i) = \sum_{j=0}^{J-1} c(i|j) P(Y_0=j | X=x_0)$$

of misclassification
the subject
to class i

$$= \sum_{j=0}^{J-1} c(i|j) \cdot \pi_j(x_0)$$

$$= \frac{\sum_{j=0}^{J-1} c(i|j) \cdot p_j \cdot f_j(x_0)}{\sum_{k=0}^{J-1} p_k \cdot f_k(x_0)}$$

$$\hat{y}_0 = \operatorname{argmin}_i ECM(i)$$

Thus $\hat{y}_0 = \operatorname{argmin}_i \left\{ \sum_{j=0}^{J-1} c(i|j) \cdot p_j \cdot f_j(x_0) \right\}$

If $c(i|j) = \begin{cases} 1 & \text{for } i \neq j \\ 0 & \text{for } i = j \end{cases}$ "0/1-loss"

This gives

$$\hat{y}_0 = \operatorname{argmin}_i \left\{ \left[\sum_{j=0}^{J-1} p_j f_j(x_0) \right] - \overbrace{p_i f_i(x_0)}^{\text{zero cost}} \right\}$$

$$= \operatorname{argmax}_i \left\{ \underline{p_i f_i(x_0)} \right\}$$

This is called Bayes classifier

