# Random number generator

From now on, we assume to have a random generator on the unit interval available.

# Simulation from discrete distributions

- Let $x$ be a stochastic variable so that $x \in \{x_1, \dots, x_k\}$, and
$$P(x = x_i) = p_i \quad \text{where} \quad \sum_{i=1}^{k} p_i = 1$$
- Define $F_i = \sum_{j=1}^{i} p_j$ for $i = 0, 1, \dots, k$
- General simulation algorithm:

  $u \sim U[0, 1]$
  **for** $i = 1, 2, \dots, k$ **do**
      **if** $u \in (F_{i-1}, F_i]$ **then**
          $x = x_i$
      **end if**
  **end for**

- Note:
  - can be used for any discrete distribution, but can be inefficient
  - more efficent searching algorithm can make the algorithm faster
  - specialised algorithm for specific distrbutions: binomial, negative binomial, poisson

# Simulation from continuous distributions

- Probability integral transform (inversion method)
- Let $x$ have density $f(x), x \in \mathbf{R}$, $F(x) = \int_{-\infty}^{x} f(z)\mathrm{d}z$
- General simulation algorithm:

  $u \sim U[0,1]$
  $x = F^{-1}(u)$
  return $x$
- We need to:
  - prove that the algorithm is correct
  - intuitively understand why it is correct
  - look at examples
  - understand when the algorithm can be used