Simulation strategies for continuous distributions

- Probability integral transform (inversion method):

$$u \in U[0,1]; \quad x = F^{-1}(u)$$

- Bivariate transformation:

$$(x_1, x_2) \sim f_x(x_1, x_2) \quad \text{and} \quad (y_1, y_2) = g(x_1, x_2)$$
$$\Rightarrow (y_1, y_2) \sim f_y(y_1, y_2) = f_x(g^{-1}(y_1, y_2)) \cdot |J|$$

- Ratio-of-uniforms method:

$$(x_1, x_2) \sim U(C_f) \quad \text{with} \quad C_f = \left\{ (x_1, x_2) \,\middle|\, 0 \le x_1 \le \sqrt{f^\star\left(\frac{x}{x_2}\right)} \right\}$$
$$\Rightarrow y = \frac{x_2}{x_1} \sim f(y) = \frac{f^\star(y)}{\int_{-\infty}^{\infty} f^\star(u)\mathrm{d}u}$$

- Scale and location parameters: f.ex.

$$x \sim N(0,1) \quad \Rightarrow \quad y = ax + b \sim N(b, a^2)$$

- Use known relations between distributions: f.ex.

$$x \sim N(0,1) \quad \Rightarrow \quad y = x^2 \sim \chi_1^2$$

Simulation strategies for continuous distributions

For each simulation strategy we have:

- Given a simulation algorithm (by pseudo-code)
- Proved that the algorithm is correct
  - produce samples from the correct distrbution
- Given example(s) of use
- Discussed intuition

Box-Muller

- Use bivariate transformation
- Box-Muller algorithm:

  $x_1 \sim U[0, 2\pi)$
  $x_2 \sim \text{Exp}\left(\frac{1}{2}\right)$
  $y_1 = \sqrt{x_2}\cos(x_1)$
  $y_2 = \sqrt{x_2}\sin(x_1)$
  return (y_1, y_2)

- Then $y_1 \sim N(0, 1)$ and $y_2 \sim N(0, 1)$, independently