Rejection sampling

- Goal: Want to sample $x \sim f(x)$ (density)
- Assume: We know how to sample $x \sim g(x)$ and we know a c so that

$$\frac{f(x)}{g(x)} \leq c \quad \text{for all } x \text{ where } f(x) > 0$$

Rejection sampling

- Goal: Want to sample $x \sim f(x)$ (density)
- Assume: We know how to sample $x \sim g(x)$ and we know a c so that

$$\frac{f(x)}{g(x)} \leq c \quad \text{for all } x \text{ where } f(x) > 0$$

- Rejection sampling algorithm:

  finished $= 0$
  while (finished $= 0$) **do**
      $x \sim g(x)$
      $\alpha = \frac{1}{c} \cdot \frac{f(x)}{g(x)}$
      $u \sim U[0, 1]$
      if $(u \leq \alpha)$ **then**
          finished $= 1$
      end if
  end while

Rejection sampling

- Goal: Want to sample $x \sim f(x)$ (density)
- Assume: We know how to sample $x \sim g(x)$ and we know a c so that

$$\frac{f(x)}{g(x)} \leq c \quad \text{for all } x \text{ where } f(x) > 0$$

- Rejection sampling algorithm:

```
finished = 0
while (finished = 0) do
    x ~ g(x)
    α = (1/c) · f(x)/g(x)
    u ~ U[0,1]
    if (u ≤ α) then
        finished = 1
    end if
end while
```

- Efficiency: $\#\text{tries} \sim \text{Geom}\left(p = \frac{1}{c}\right)$ so $E[\#\text{tries}] = \frac{1}{p} = c$
- The art of rejection sampling is to find a $g(x)$ that is similar to $f(x)$ and which we know how to sample from.

Adaptive rejection sampling idea

RS algorithm:

finished $= 0$
while (finished $= 0$) **do**
  $x \sim g(x)$
  $\alpha = \frac{1}{c} \cdot \frac{f(x)}{g(x)}$
  $u \sim U[0,1]$
  if $(u \leq \alpha)$ **then**
    finished $= 1$
  end if
end while

Modified RS algorithm:

finished $= 0$
$i = 0$
while (finished $= 0$) **do**
  $i = i + 1$
  $x \sim g_i(x)$
  $\alpha = \frac{1}{c_i} \cdot \frac{f(x)}{g_i(x)}$
  $u \sim U[0,1]$
  if $(u \leq \alpha)$ **then**
    finished $= 1$
  end if
end while

Adaptive rejection sampling idea

RS algorithm:

finished $= 0$
while (finished $= 0$) **do**
    $x \sim g(x)$
    $\alpha = \frac{1}{c} \cdot \frac{f(x)}{g(x)}$
    $u \sim U[0,1]$
    if $(u \leq \alpha)$ **then**
        finished $= 1$
    end if
end while

Modified RS algorithm:

finished $= 0$
$i = 0$
while (finished $= 0$) **do**
    $i = i + 1$
    $x \sim g_i(x)$
    $\alpha = \frac{1}{c_i} \cdot \frac{f(x)}{g_i(x)}$
    $u \sim U[0,1]$
    if $(u \leq \alpha)$ **then**
        finished $= 1$
    end if
end while

- Question: How should we choose $g_i(x)$ so that $g_i(x)$ is becoming more similar to $f(x)$ when i increases?