

Improper priors

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★ Can we use

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as our prior?

★ It is not a distribution!! Is an example of an improper prior

★ Ignoring that this prior is not a distribution, the corresponding posterior

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- ★ Improper priors are often used
 - but then you need to check that the posterior is proper!!
 - with a proper prior the posterior is always proper

Non-informative priors

- ★ Have data x and likelihood $f(x|\theta)$
- ★ What prior to use if we have no prior information about θ ?
- ★ Different strategies has been proposed
- ★ Jeffrey's non-informative prior:

$$f(\theta) \propto \sqrt{I(\theta)}$$

where $I(\theta)$ is the Fisher information,

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- ★ Some examples of Jeffrey's priors:
 - θ is location parameter, $f(x|\theta) = g(x - \theta)$: $f(\theta) \propto c$
 - θ is scale parameter, $f(x|\theta) = g(x/\theta)/\theta$: $f(\theta) \propto \frac{1}{\theta}$

