Plug in estimator for the skewness param

$$\theta = \frac{E(X^3) - 3\mu\cdot\sigma^2 - \mu^2}{\sigma^3}$$

$$\hat{\theta} = \frac{E_{\hat{F}}(X^3) - 3E_{\hat{F}}(X)\cdot Var_{\hat{F}}(X) - [E_{\hat{F}}(X)]^2}{[Var_{\hat{F}}(X)]^{3/2}} =$$

$$= \frac{\frac{1}{n}\sum X_i^3 - 3\frac{1}{n}\cdot\sum X_i \cdot \frac{1}{n}\sum (X_i - \bar{X})^2 - \left[\frac{1}{n}\sum X_i\right]^2}{\left[\frac{1}{n}\sum (X_i - \bar{X})^2\right]^{3/2}}$$

- No ~~as-param~~ distributional assumptions
- one way of building an estimator

Real world

$F \xrightarrow{iid} X_1, \dots, X_n \Longrightarrow$

$\theta = t(F)$

$\hat{\theta} = s(X)$

Bootstrap world

$\hat{F} \longrightarrow (X_1^*, \dots, X_n^*)$

$\hat{\theta}^* = s(X^*)$

We get $\hat{\theta}^*(1) \ldots \hat{\theta}^*(B)$ this is the basis for estimating the distribution of $\hat{\theta}$

- Estimate the $SD_F(\hat{\theta})$ as

$$SD_{\hat{F}}(\hat{\theta}^*) = \frac{1}{B-1} \sum_{b=1}^{B} (\hat{\theta}^*(b) - \hat{\theta}^*(\cdot))^2$$

where $\hat{\theta}^*(\cdot) = \frac{1}{B} \sum \hat{\theta}^*(b)$

- Estimate the bias

$$bias_F(\hat{\theta}, \theta) = E_F(\hat{\theta}) - t(F)$$

$$\hat{bias}_{\hat{F}}(\hat{\theta}^*, \hat{\theta}) = E_{\hat{F}}(\hat{\theta}^*) - t(\hat{F})$$

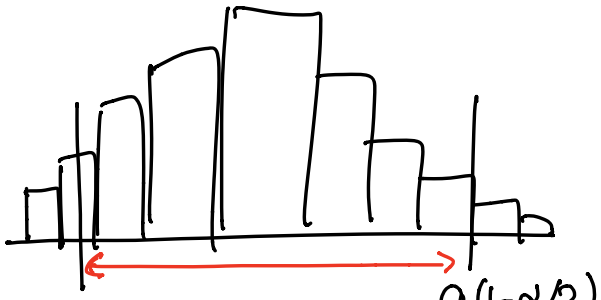
$$E_{\hat{F}}(\hat{\theta}^*) = \frac{1}{B} \sum \hat{\theta}^*(b)$$

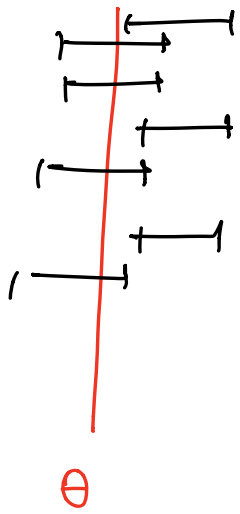
Bias corrected estimate

$$\underline{\hat{\theta}_c} = \hat{\theta} - \left[E_{\hat{F}}(\hat{\theta}^*) - t(\hat{F}) \right]$$

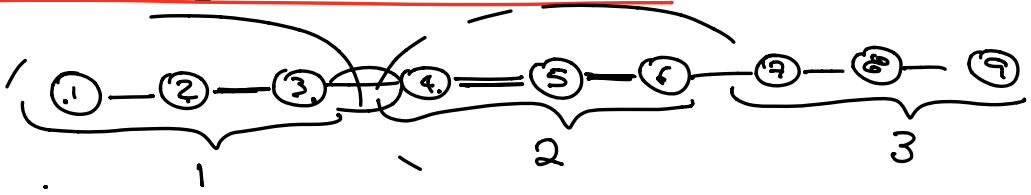
$$Var(\hat{\theta}_c) > Var(\hat{\theta})$$

- Estimate conf. intervals

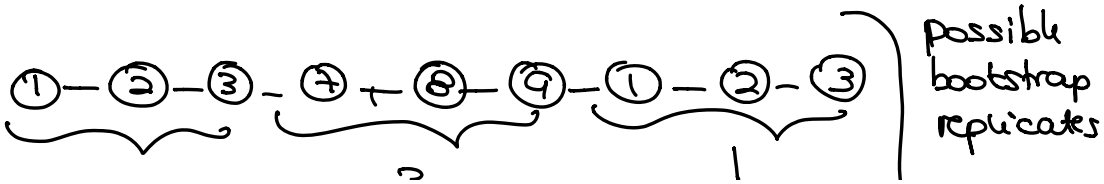


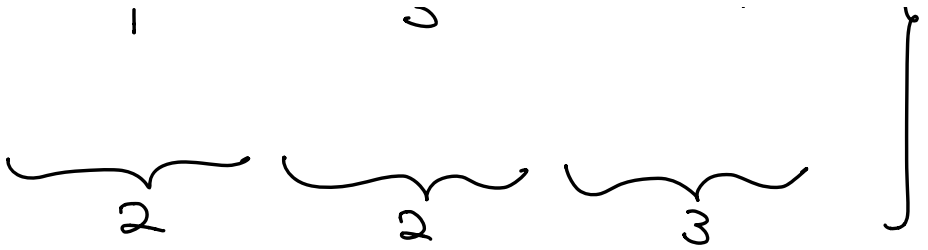


Non-moving blocks bootstrap

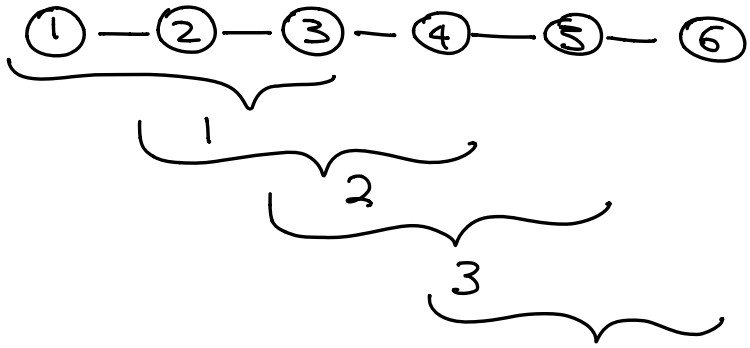


resample the blocks with replacement





Moving blocks



Resample blocks with replacement

Block of Blocks

$$X = (X_1, X_2, \ldots, X_n)$$

$$Y = \left[\begin{matrix} (X_1)(X_2)(X_3) & \cdots & (X_{n-1}) \\ (X_2)(X_3)(X_4) & \cdots & (X_n) \end{matrix} \right] \begin{matrix} Y_1 \\ Y_2 \end{matrix}$$

- Stat of interest can be written as a function of Y

Lag 1 cov $\Rightarrow$ $cov(Y(1), Y(2))$