Random walk model of order 1 (RW1)

Parametrization

The random walk model of order 1 (RW1) for the Gaussian vector \( x = (x_1, \ldots, x_n) \) is constructed assuming independent increments:

\[
\Delta x_i = x_i - x_{i+1} \sim \mathcal{N}(0, \tau^{-1})
\]

The density for \( x \) is derived from its \( n-1 \) increments as

\[
\pi(x | \tau) \propto \tau^{(n-1)/2} \exp \left\{ -\frac{\tau}{2} \sum \Delta x_i^2 \right\} = \tau^{(n-1)/2} \exp \left\{ -\frac{1}{2} x^T Qx \right\}
\]

where \( Q = \tau R \) and \( R \) is the structure matrix reflecting the neighbourhood structure of the model.

It is also possible to define a cyclic version of the RW1 model, in this case the graph is modified so that last node \( x_n \) is neighbour of \( x_{n-1} \) and \( x_1 \).

Hyperparameters

The precision parameter \( \tau \) is represented as

\[
\theta = \log \tau
\]

and the prior is defined on \( \theta \).

Specification

The RW1 model is specified inside the \( f() \) function as

\[
f(<\text{whatever}>, \text{model}="\text{rw1}" , \text{values}=<\text{values}>, \text{cyclic}=<\text{TRUE}|\text{FALSE}>,
\]

\[
\text{hyper} = <\text{hyper}>, \text{scale.model} = \text{FALSE}
\]

The (optional) argument values is a numeric or factor vector giving the values assumed by the covariate for which we want the effect to be estimated. See next example for an application.

The logical option scale.model determine if the model should be scaled to have an average variance (the diagonal of the generalized inverse) equal to 1. This makes prior spesification much easier. Default is FALSE so that the model is not scaled.

Current recommended prior

If you do not know which prior to use, the current recommendation is

\[
u = 1
\]

\[
f(<\text{whatever}>, \text{model}="\text{rw1}" , \text{scale.model} = \text{TRUE}
\]

\[
\text{hyper} = \text{list(theta} = \text{list(prior="pc.prec", param=c(u,0.01)))}
\]

\[
inla.doc("\text{pc.prec}"
\]

where \( u \) should be set to a value appropriate for your case:

**Gaussian likelihood (no link)** Set \( u \) to be the empirical standard deviation of your data

**Poisson likelihood and log link** Set \( u \) to 1

**Binomial and logit link** Set \( u \) to 0.5

**Binomial and probit link** Set \( u \) to 0.33

Increasing \( u \) gives a weaker prior, decreasing \( u \) gives a stronger prior.
Hyperparameter specification and default values

hyper

theta

  hyperid 4001
  name log precision
  short.name prec
  prior loggamma
  param 1 5e-05
  initial 4
  fixed FALSE
  to.theta function(x) log(x)
  from.theta function(x) exp(x)

constr TRUE

nrow.ncol FALSE

augmented FALSE

aug.factor 1

aug.constr

n.div.by

n.required FALSE

set.default.values FALSE

min.diff 1e-05

pdf rw1

Example

n=100
z=seq(0,6,length.out=n)
Y=sin(z)+rnorm(n,mean=0,sd=0.5)
data=data.frame(y=Y,z=z)

formula=y~f(z,model="rw1",
        hyper = list(prec = list(prior="loggamma",param=c(1,0.01))))
result=inla(formula,data=data,family="gaussian")

Notes

- The RW1 is intrinsic with rank deficiency 1.
- The RW1 model for irregular locations are supported although not described here.
- The term $\frac{1}{2} \log(|R|^*)$ of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it.