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Lithology-Fluid Inversion based on Prestack Seismic Data

Marit Ulvmoen

Summary

The focus of the study is on lithology-fluid inversion from prestack seismic data. The target zone is a 3D reservoir, and the inversion problem is solved in a Bayesian framework where the complete solution is given by the posterior model. The likelihood model relates the lithology-fluid classes to elastic variables and the seismic data, and it follows the lines of Larsen et al. (2006). In order to make allowances to the strong lateral coupling between the lithology-fluid classes, the prior model is defined as a profile Markov random field. To model vertical continuity of the lithology-fluid classes along the profiles, a Markov chain model upward through the reservoir is used. The posterior model is given as the complete set of the full conditional pdf's in the profile Markov random field model, and a block Gibbs simulation algorithm is used laterally. The profiles are simulated exactly using the efficient upward-downward algorithm defined in Larsen et al. (2006). The inversion model is evaluated on a synthetic 2D reservoir. The lithology-fluid classes in the reference model have strong horizontal continuity with thin layers of shale, and the fully coupled 3D model provides reliable results.

Introduction

Lithology-Fluid (LF) inversion from seismic data is important in exploration and development of petroleum reservoirs. The inverse problem is ill-posed, such that several sets of LF classes may result in the same seismic data. The objective of the study is to map LF classes in a 3D reservoir, and a Bayesian approach is used. The lithologies considered are shale and sandstone, and the sandstone is saturated with one of the fluids gas, oil or brine, but other LF classes may also be of interest. The LF classes are denoted $\boldsymbol{\pi} : \{\pi_{\boldsymbol{x},t}; (\boldsymbol{x}, t) \in \mathcal{L}_\mathcal{D}\}$ where $\mathcal{L}_\mathcal{D}$ is a discretization of the reservoir in lateral positions $\boldsymbol{x} \in \mathcal{L}_\mathcal{D}^{\boldsymbol{x}}$ corresponding to inline and xline positions, and in time $t \in \{1, \dots, T\} \in \mathcal{L}_\mathcal{D}^t$ downward. The inversion is performed from seismic prestack data $\boldsymbol{d}$ for a set of reflection angles. In order to link the LF classes and the seismic data, the elastic variables P-wave velocity, S-wave velocity and density are used. The log-transform of the elastic variables is denoted by $\boldsymbol{m} : \{m_{\boldsymbol{x},t}; (\boldsymbol{x}, t) \in \mathcal{L}_\mathcal{D}\}$.

Stochastic Model

The inversion is performed in a Bayesian setting, where the complete solution is given by the posterior model

$$p(\boldsymbol{\pi}|\boldsymbol{d}) = \text{const} \times p(\boldsymbol{d}|\boldsymbol{\pi})\, p(\boldsymbol{\pi})$$

where $p(\boldsymbol{d}|\boldsymbol{\pi})$ is the likelihood model, $p(\boldsymbol{\pi})$ is the prior model and const is a normalizing constant which is difficult to determine. From the posterior model, we obtain the locationwise most probable solution $\hat{\boldsymbol{\pi}}$ and realizations of $\boldsymbol{\pi}$.

Likelihood Model

The likelihood model defines the likelihood of the LF classes $\boldsymbol{\pi}$ given the seismic data $\boldsymbol{d}$. In order to link the LF classes and the seismic data, the likelihood model is decomposed into

$$p(\boldsymbol{d}|\boldsymbol{\pi}) = \int \dots \int p(\boldsymbol{d}|\boldsymbol{m})\, p(\boldsymbol{m}|\boldsymbol{\pi})\, d\boldsymbol{m}$$

where $p(\boldsymbol{d}|\boldsymbol{m})$ is a seismic response likelihood model and $p(\boldsymbol{m}|\boldsymbol{\pi})$ is a rock physics likelihood model. The rock physics likelihood model has no spatial dependence, and is factorized into

$$p(\boldsymbol{m}|\boldsymbol{\pi}) = \prod_{\boldsymbol{x}} \prod_{t} p(\boldsymbol{m}_{\boldsymbol{x},t}|\pi_{\boldsymbol{x},t}).$$

The seismic response likelihood model is defined from a vertical convolutional model $\boldsymbol{d} = \boldsymbol{s} + \boldsymbol{e} = \boldsymbol{G}\boldsymbol{m} + \boldsymbol{e}$, where $\boldsymbol{s}$ is the seismic signal, $\boldsymbol{e}$ is observation error and $\boldsymbol{G}$ is a modeling matrix defined by $\boldsymbol{G} = \boldsymbol{W}\boldsymbol{A}\boldsymbol{D}$ where $\boldsymbol{W}$ is a block-diagonal matrix containing one wavelet for each reflection angle, $\boldsymbol{A}$ is a matrix of angle-dependent weak contrast Aki-Richards coefficients and $\boldsymbol{D}$ is a differential matrix giving the contrasts of the log-transforms of the elastic properties. The seismic response likelihood model is factorized

$$p(\boldsymbol{d}|\boldsymbol{m}) = \text{const} \times \frac{p_*(\boldsymbol{m}|\boldsymbol{d})}{p_*(\boldsymbol{m})}$$

where $p_*(\boldsymbol{m})$ and $p_*(\boldsymbol{m}|\boldsymbol{d})$ are Gaussian prior and posterior pdf's for linearized Zoeppritz AVO inversion, see Buland and Omre (2003).

Prior Model

The horizontal coupling between the LF classes in an earth model is very strong, and in order to make allowances to this coupling we let the field follow a profile Markov random field given by

$$p(\boldsymbol{\pi}_{\boldsymbol{x}}|\boldsymbol{\pi}_{-\boldsymbol{x}}) = p(\boldsymbol{\pi}_{\boldsymbol{x}}|\boldsymbol{\pi}_{\boldsymbol{y}}; \boldsymbol{y} \in \delta(\boldsymbol{x}));\ \text{all } \boldsymbol{x} \in \mathcal{L}_{\mathcal{D}}^{\boldsymbol{x}}$$

where $\boldsymbol{\pi}_{\boldsymbol{x}} : \{\pi_{\boldsymbol{x},t}; t \in \mathcal{L}_{\mathcal{D}}^{t}\}$ is a vertical profile in an arbitrary $\boldsymbol{x}$ in $\mathcal{L}_{\mathcal{D}}^{\boldsymbol{x}}$, $\boldsymbol{\pi}_{-\boldsymbol{x}} : \{\boldsymbol{\pi}_{\boldsymbol{y}}; \boldsymbol{y} \in \mathcal{L}_{\mathcal{D}}^{\boldsymbol{x}}, \boldsymbol{y} \neq \boldsymbol{x}\}$ is the set of all LF profiles except $\boldsymbol{\pi}_{\boldsymbol{x}}$ and $\delta(\boldsymbol{x})$ is a fixed neighbourhood of $\boldsymbol{x}$ in $\mathcal{L}_{\mathcal{D}}^{\boldsymbol{x}}$. Hence, given the profiles in the neighbourhood $\delta(\boldsymbol{x})$, the LF profile $\boldsymbol{\pi}_{\boldsymbol{x}}$ is independent of the rest of the field. According to the Hammersley-Clifford theorem, the set of all conditional pdf's $p(\boldsymbol{\pi}_{\boldsymbol{x}}|\boldsymbol{\pi}_{-\boldsymbol{x}})$ fully specify the prior model $p(\boldsymbol{\pi})$ as a Markov random field, see Besag (1974).

Each profile $\boldsymbol{\pi}_{\boldsymbol{x}}$ follows a Markov chain model upwards through the target zone like in Larsen et al. (2006), expressed

$$p(\boldsymbol{\pi}_{\boldsymbol{x}}|\boldsymbol{\pi}_{\boldsymbol{y}}; \boldsymbol{y} \in \delta(\boldsymbol{x})) = \prod_{t} p(\pi_{\boldsymbol{x},t}|\pi_{\boldsymbol{x},t+1}, \pi_{\boldsymbol{y},t}; \boldsymbol{y} \in \delta(\boldsymbol{x}));\ \text{all } \boldsymbol{x} \in \mathcal{L}_{\mathcal{D}}^{\boldsymbol{x}}.$$

This entails that the conditional pdf of LF class $\pi_{\boldsymbol{x},t}$ given the LF class immediately below and the LF classes at t in $\delta(\boldsymbol{x})$ is independent of the rest of the field. The full conditional pdf $p(\pi_{\boldsymbol{x},t}|\pi_{-(\boldsymbol{x},t)})$ is only a function of the LF classes immediately above and below, in addition to the LF classes at t in $\delta(\boldsymbol{x})$, hence $\boldsymbol{\pi}$ is a Markov random field althoug with an unusual parametrization.

Posterior Model

The posterior pdf is completely determined by the likelihood and prior models, and it is given as

$$p(\boldsymbol{\pi}|\boldsymbol{d}) = \text{const} \times \left[\int \cdots \int \frac{p_*(\boldsymbol{m}|\boldsymbol{d})}{p_*(\boldsymbol{m})}\, p(\boldsymbol{m}|\boldsymbol{\pi})\, d\boldsymbol{m}\right] p(\boldsymbol{\pi}) = \text{const} \times l(\boldsymbol{d}|\boldsymbol{\pi})\, p(\boldsymbol{\pi})$$

where $l(\boldsymbol{d}|\boldsymbol{\pi})$ is the likelihood model including a high dimensional integral over the three elastic variables over the target zone. To avoid the high dimensional integral, an approximation of the posterior pdf is constructed such that the likelihood model factorizes. In the approximation, spatial correlation in the pdf's $p_*(\boldsymbol{m})$ and $p_*(\boldsymbol{m}|\boldsymbol{d})$ is ignored, and the approximate likelihood model can be written

$$\tilde{l}(\boldsymbol{d}|\pi_{\boldsymbol{x},t}) = \int\int\int \frac{p_*(\boldsymbol{m}_{\boldsymbol{x},t}|\boldsymbol{d})}{p_*(\boldsymbol{m}_{\boldsymbol{x},t})}\, p(\boldsymbol{m}_{\boldsymbol{x},t}|\boldsymbol{\pi}_{\boldsymbol{x},t})\, d\boldsymbol{m}_{\boldsymbol{x},t}.$$

This integral is of dimension three, and numerically tracable. The prior model follows a profile Markov random field model, and with a likelihood model that factorizes, the associated conditional posterior pdf's can be written

$$\tilde{p}(\boldsymbol{\pi}_{\boldsymbol{x}}|\boldsymbol{\pi}_{-\boldsymbol{x}}, \boldsymbol{d}) = \text{const} \times \prod_{t} \tilde{l}(\boldsymbol{d}|\pi_{\boldsymbol{x},t})\, p(\pi_{\boldsymbol{x},t}|\pi_{\boldsymbol{x},t+1}, \pi_{\boldsymbol{y},t}; \boldsymbol{y} \in \delta(\boldsymbol{x}))\ ;\ \text{all } \boldsymbol{x} \in \mathcal{L}_{\mathcal{D}}^{\boldsymbol{x}}.$$

Given the profiles in the neighbourhood $\delta(\boldsymbol{x})$ and the seismic data $\boldsymbol{d}$, the LF profiles $\boldsymbol{\pi}_{\boldsymbol{x}}$ are independent of the rest of the field. Hence, the set of all conditional pdf's

$\tilde{p}(\boldsymbol{\pi}_{\boldsymbol{x}}|\boldsymbol{\pi}_{-\boldsymbol{x}}, \boldsymbol{d})$ fully specify the 3D posterior model for lithology-fluid inversion $\tilde{p}(\boldsymbol{\pi}|\boldsymbol{d})$ as a Markov random field.

As the profile Markov random field is fully specified by the complete set of all conditional posterior pdf's $\tilde{p}(\boldsymbol{\pi}_{\boldsymbol{x}}|\boldsymbol{\pi}_{-\boldsymbol{x}}, \boldsymbol{d})$, a block Gibbs simulation algorithm may be used laterally. The conditional posterior pdf's are on the same form as in Larsen et al. (2006), hence the efficient recursive upward-downward algorithm defined in this paper can be used to simulate from the conditional posterior pdf's exactly. The algorithm is initiated in an arbitrary configuration of $\boldsymbol{\pi}$. Then, in each iteration, a position $\boldsymbol{x}$ is drawn uniformly from $\mathcal{L}_{\mathcal{D}}^{\boldsymbol{x}}$ and the profile $\boldsymbol{\pi}_{\boldsymbol{x}}$ is generated from $\tilde{p}(\boldsymbol{\pi}_{\boldsymbol{x}}|\boldsymbol{\pi}_{-\boldsymbol{x}}, \boldsymbol{d})$ by the upward-downward simulation algorithm. The algorithm converges such that $\boldsymbol{\pi}$ will be a sample from $\tilde{p}(\boldsymbol{\pi}|\boldsymbol{d})$. Note that although the model is defined in 3D, the iterative Gibbs simulation algorithm only operates in 2D with the third dimension simulated from the extremely fast recursive upward-downward algorithm. Hence the problem of slow convergence in 3D McMC algorithms is avoided.

Results and Conclusions

In order to evaluate the inversion model, a 2D reference reservoir is made, and synthetic seismic data are generated, see Figure 1.

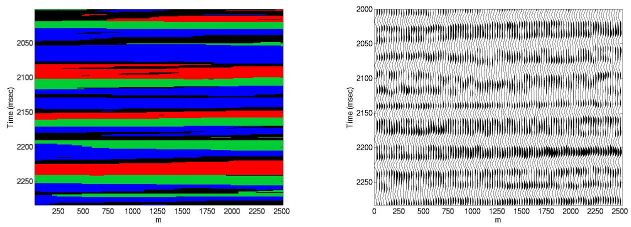

Figure 1: Reference LF characteristics $\boldsymbol{\pi}$ with gas-saturated sandstone (red), oil-saturated sandstone (green), brine-saturated sandstone (blue) and shale (black); and synthetic stacked seismic data $\boldsymbol{d}$ from the angles $\boldsymbol{\theta} = (0, 10, 20, 30, 40)$ degrees.

Figure 2 contains a set of samples for the elastic variables given the LF classes of consideration, from which the rock physics likelihood model is calculated.

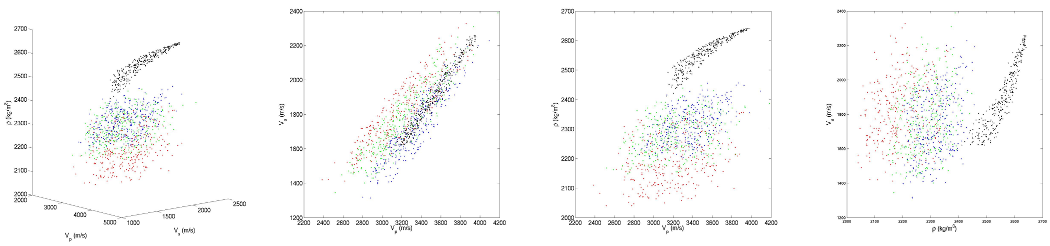

Figure 2: Elastic variables $\boldsymbol{m}$ represented by P-wave velocity (V_p), S-wave velocity (V_s) and density (ρ) given gas-saturated sandstone (red), oil-saturated sandstone (green), brine-saturated sandstone (blue) and shale (black) simulated from a rock physics model.

Figure 3 contains three independent realizations of LF characteristics generated from the approximate posterior pdf $\tilde{p}(\boldsymbol{\pi}|\boldsymbol{d})$. The realizations have realistic heterogeneity. They span the prediction uncertainty, and can be considered as possible LF characteristics.

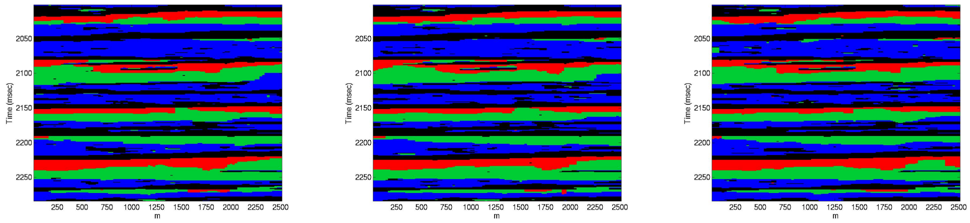

Figure 3: Independent realizations of LF characteristics from approximate posterior pdf $\tilde{p}(\boldsymbol{\pi}|\boldsymbol{d})$.

Figure 4 contains the locationwise most probable solution $\hat{\boldsymbol{\pi}}$ from the inversion model with three different prior models. The first model is the one described in the study. The second model ignores spatial dependence between neighbouring profiles. The third model ignores all dependence between the nodes. The structure in the first model is the same as in the reference reservoir. In the second model the fluid segregation condition is not fullfilled as the geology in the solution could not have been observed in nature. The third model tends to overclassify shale, and the solution is not realistic with respect to fluid segregation.

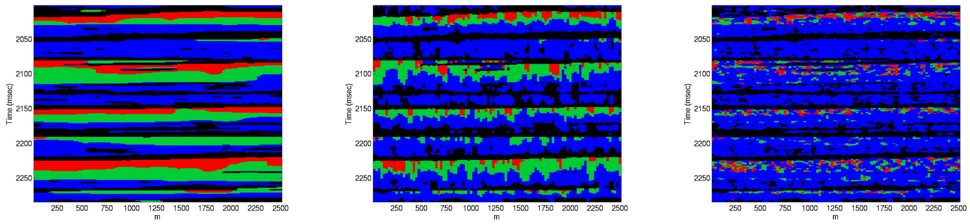

Figure 4: Marginal map LF characteristics prediction $\hat{\boldsymbol{\pi}}$; marginal map LF characteristics prediction from model where dependence between the profiles is ignored; and marginal map LF characteristics prediction from model without spatial coupling.

It is clear that if the LF classes have strong horizontal continuity, a fully coupled 3D model as prescribed here provides more reliable results than non-spatial models. In the presence of well-data, the 3D model will be of even greater importance.

References

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