A (n, N) maintenance strategy with non-periodic inspections for a multi-component system

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Abstract

In this communication, a condition-based maintenance policy and its associated cost model are proposed to tackle with the economic dependencies in multicomponent system. The proposed policy allows to determine simultaneously the current operations and the next intervention date as a function of the overall system state. Numerical examples are discussed to underline the necessity of grouping maintenance operations according to the maintenance data.

1 Introduction and problem statement

This communication considers the maintenance of a technical device or a structure composed of several items subject to a continuous gradual random deterioration. The modelling and the optimization of the maintenance for multi-component systems can rapidly lead to complex problems which can not be solved by the approaches used for mono-component systems. Indeed, various dependencies between the system's components have to be taken into account in the global maintenance decision process : e.g. the deterioration behaviors of the components can be linked together through stochastic dependencies or some of maintenance tasks have to be performed simultaneously on several entities of the system. In this communication, we consider only the existence of economic dependencies between components, i.e. costs savings can be earned when several components are maintained together rather than separately (economies of scale).

Numerous multi-component maintenance models have been developed to allow to group together maintenance tasks, but few of them have been proposed in the context of condition-based maintenance (Dekker, van der Duyn Schouten, and Wildeman 1997; van der Duyn Schouten 1996). The promising performance of condition-based maintenance policies for single-component systems leads to base the maintenance decision rules for the whole system on the knowledge of the deterioration levels of each component. Furthermore, we choose a maintenance model in the class of the control-limit policies for which the maintenance decision is made by comparing the deterioration levels to critical thresholds. We extend the classical (n, N) replacement policy presented in (van der Duyn Schouten and Vanneste 1990) for which the state of the system is continuously observed to a decision rule which allows aperiodic inspections intervals. Knowing the deterioration state of a system's component and the maintenance actions made on the rest of the system, the decision rule should allow to determine the nature of the current operation on this component (replacement or left as it is) and the date of the next inspection of the system.

The objective of the work presented in this paper is to build a parametric structure of condition-based maintenance for multi-component systems and to develop the associated cost model. The cost model can be used to assess and to optimize the performance of the policy. The optimization of the maintenance parameters gives the best compromise between preventive and corrective maintenances and allows to control the influences of the operation groupings in order to reduce the individual operating cost for each component and the global average cost of exploitation. This paper is an analytical development of a problem studied by stochastic simulations (Bérenguer, Grall, and Castanier 2000).

2 Maintenance decision framework and associated cost model

For each component i of the system, we consider a condition-based maintenance rule based on a multi-threshold structure defining several regions for the maintenance decisions : inspection, preventive and corrective replacement (Castanier, Bérenguer, and Grall 2001). When the measured deterioration state of the component i falls in its own “preventive replacement” region, this component is preventively replaced, and the next inspection is scheduled. At these decision thresholds $\xi_k^{(i)}$, $k = 1, \cdots, n_i$, we add another threshold ζ_i defining the zone of “forced maintenance”. When a replacement is made on one of the system’s components, a replacement is also simultaneously performed on each of the components whose deterioration level belongs to its respective forced maintenance zone (Castanier 2001). The inspection decision rule is defined as follows:

- if the measured deterioration level of the component i belongs to a decision zone $[\xi_k^{(i)}, \xi_{k+1}^{(i)})$, $k = 0, \cdots, n_i - 1$, the next inspection date of this component is scheduled $n_i - k$ time periods later,
- the system is completely inspected as soon as an inspection of at least one component is realized. So the next global system inspection date is the smallest date given among all the dates scheduled for every component. Finally, by prolonging this reasoning, we can prove that every monocomponent policy should have the same number of thresholds. Note that the decision parameter set is so defined by $\xi_k^{(i)}$, $k = 1, \cdots, n$ and ζ_i whith $n = n_1 = n_2$ for a two-identical components system.

The behaviour of the two-component system state subject to a multi-threshold policy ($n = 2$) can be illustrated by Figure 1. The cost of a maintenance operation consists of a specific unit cost (c_i for an inspection, $c_p^{(i)}$ (resp. $c_c^{(i)}$) for a preventive (resp. corrective) replacement of the component i) and a set-up cost C_s for the maintenance which is incurred only once in case of multiple operations performed simultaneously. To assess the performance of the proposed maintenance policy, a long-run average cost criterion of the system exploitation is constructed including the cost incurred by maintenance operations. The evaluation of such a criterion requires the determination of the stationary laws associated to the behavior of the maintained system. After a complete replacement of the system (all the components are replaced), the system is as good as new. These replacement times are regenerative points for the stochastic process describing the maintained system state. Beyond its regenerative characteristics, the semi-regenerative properties of the stochastic process describing the maintained system state (Cocozza-

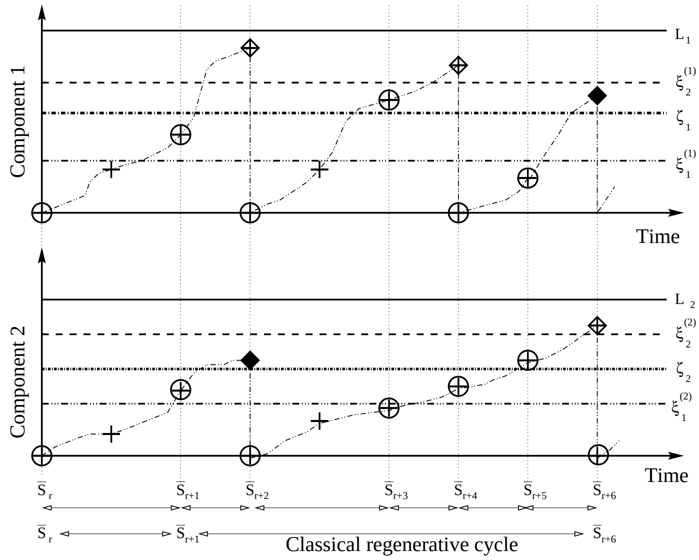

Figure 1: Deterioration evolution of a two-component system subject to a multi-threshold policy - $\bar{S}_r$: successive intervention dates on components

Thivent 2000) allow us to restrict the study to a semi-regenerative cycle defined by the time-interval between any two successive maintenance operations. On such a cycle, the evolution of the maintained system states can be easily determined from different simpler maintenance scenarios. Finally, the expression of the long-run average cost of the system exploitation per unit of time C_∞ can be written as the following ratio:

$$C_\infty = \lim_{t\to\infty} \frac{C(t)}{t} = \frac{E_\pi(C(\bar{S}_r - \bar{S}_{r-1}))}{E_\pi(\bar{S}_r - \bar{S}_{r-1})} \tag{1}$$

At steady-state, it is possible to construct the stationary probability density of the maintained system state π and so to determine the different expectations $E_\pi(C(\bar{S}_r - \bar{S}_{r-1}))$ and $E_\pi(\bar{S}_r - \bar{S}_{r-1})$ which are respectively the average cumulated cost in a semi-regenerative cycle and the average number of time periods elapsed during two successive interventions dates $\bar{S}_r$.

3 Numerical results and conclusions

To illustrate the performance of the policy, we propose to discuss about the numerical results proposed in Figures 2 and 3. Figures 2 and 3 represent long-run average cost curves obtained by variations of the two-“forced maintenance” thresholds ζ_i, $i = 1, 2$ from 0 to their respective preventive replacement threshold $\xi_n^{(i)}$ for different set-up cost values C_s and component deterioration characteristics. For each monocomponent policy, the parameters $\xi_k^{(i)}$, $k = 1, \cdots, n$ are optimized without taking into account possible operation groupings. Let C_{opt} be the minimal average cost for the optimal threshold values ζ_1^* and ζ_2^*. Let C_{ind} be the sum of the costs obtained with two optimized monocomponent policies without taking into account possible operation groupings. We note beforehand that C_{opt} remains lower than C_{ind}. The increase of the number of operation groupings (Figures 2) is entailed by the increase of the set-up cost values. Finally, when the set-up cost is either very weak (Figure 2 left) or very strong (Figure 2 right), the two extreme following cases are observable:

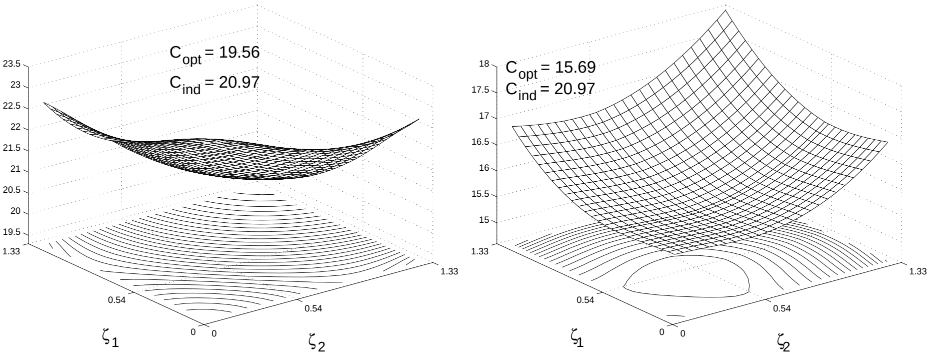

Figure 2: Maintenance cost surfaces for a system with two identical components and equivalent unit operation costs and different set-up costs - Parameters: $L = 2$, $c_i = 1$, $c_p = 40$, $c_c = 100$ ($C_s = 0$ on left-hand side and $C_s = 30$ on right-hand side)

- $\zeta_1^* = \xi_n^{(1)}$ and $\zeta_2^* = \xi_n^{(2)}$ which results from a very weak economic dependence. This resulting policy can be seen as the simple combination of the two monocomponent policies.
- $\zeta_1^* = \zeta_2^* \approx 0$ which results from a strong economic dependence. The system is replaced as soon as a component requires a replacement and so the evolutions of every component are forced to be identical (same operations at the same time).

Figures 3 illustrate the evolution of the maintenance cost surfaces for two non identical components (one fast- and one slow-deteriorating component). Unit costs and set-up costs for these two studies are constant and only deterioration characteristics vary. When these deterioration characteristics are similar (Figure 3 left), the maintenance is naturally similar on both components ($\zeta_1 \approx \zeta_2$). When these

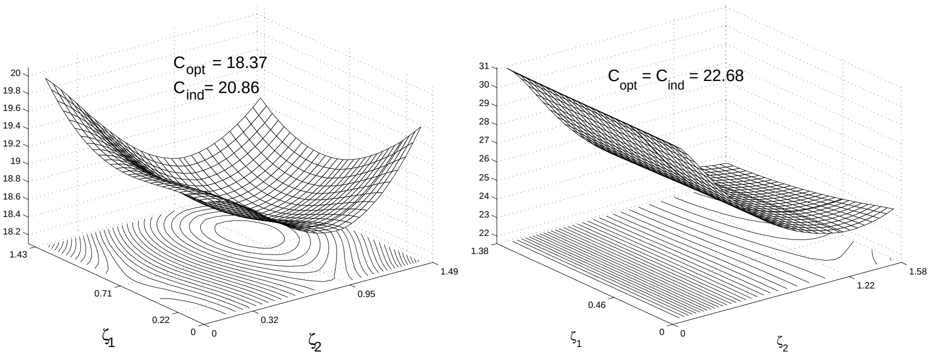

Figure 3: Maintenance cost surfaces for a system with two different components and equivalent unit operation costs (one fast- and slow-deteriorating component: not far away speeds on the left-hand side and very different speeds on the right-hand side) - Parameters: $L = 2$, $c_i = 1$, $c_p = 50$, $c_c = 100$, $C_s = 20$

characteristics are very different (Figure 3 right), the valley-shaped cost surface illustrates the strong influence of the forced-maintenance threshold of the slowier deteriorating component ζ_2 on the average cost value. In fact, only the evolution of the fast-deteriorating component (component 1) leads to the replacement-grouping decision: a high-frequency maintenance planning is required opposed to the slow-deteriorating component.

To conclude, these numerical examples underline the necessity of controlling the operation groupings. Basing this grouping decision on the knowledge of each component state offers an efficient tool to optimize the compromise between forced-maintenance (a maintenance operation is anticipated on a component to realize two simultaneous operations) and the maintenances realized separately. Nevertheless, the increase of the components number entails the increase of the decision parameters number to optimize as well as the associated numeric difficulties.

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