Considering Prior Information for Accelerated Tests with a Lifetime-Ratio

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Abstract

The higher the reliability requirements on a product are, the more extensive the test must be to ensure the required reliability. One is often unable to obtain a reasonable amount of test items, when stresses approximate normal operating conditions. To reduce sample-sizes and test duration components are exposed to much higher stresses during the test than within their normal use conditions. Therefore, it has to be ensured that the failure mechanism of the component operating under higher stress is identical to the failure mechanism under normal stress. Then, the failure behaviour under normal operating conditions can be predicted if the value of the acceleration factor is known. The so-called lifetime-ratio considers a case where the test time is not equal to the required product lifetime. Furthermore, there exist different *Bayesian* methods to reduce the test effort. In this paper the acceleration factor is added to the methods by *Kleyner* et al. and *Beyer/Lauster*. Furthermore, the lifetime-ratio is added to the method by *Kleyner* et al.. The methods are compared and discussed within a practical example where the required reliability for a mechanical component has to be ensured.

1. Acceleration Factor and Lifetime-Ratio

For the prediction of the component failure behaviour under normal operating conditions it is necessary to know the value of the acceleration factor. The acceleration factor shows how high the lifetime t_s of a component is under normal operating conditions in relation to the component lifetime t_t under highly stressed conditions. The acceleration factor is given by (*Tobias 1998*)

$$\chi = \frac{t_s}{t_t}\,. \tag{1}$$

The usual description of the failure behaviour of mechanical systems or components is provided by the three-parameter *Weibull* distribution (e.g. (*Bertsche 1999*)), with the pdf

$$f(t) = \frac{b}{(T-t_0)}\left(\frac{t-t_0}{T-t_0}\right)^{b-1} e^{-\left(\frac{t-t_0}{T-t_0}\right)^{b}},\quad 0 \le t_0 \le t \tag{2}$$

with the scale parameter T, the shape parameter b and the location parameter t_0. For the two-parameter *Weibull* distribution, with the location parameter $t_0 = 0$, the reliability for the accelerated test is given by the survival function

$$R_t(t) = e^{-\left(\frac{t}{T_t}\right)^{b_t}}\,. \tag{3}$$

The survival function for the component under normal operating conditions is

$$R_s(t) = e^{-\left(\frac{t}{T_s}\right)^{b_s}}\,. \tag{4}$$

With the assumptions $b_t = b_s = b$ and $\chi T_t = T_s$ and setting eq. (3) equal to eq. (4) one obtains the following condition

$$R_t(t) = R_s(t)^{\chi^b}\,. \tag{5}$$

For a case where the test time t_t is not equal to the required product lifetime t_s it is necessary to introduce the so-called lifetime-ratio that is defined as (*Beyer 1990*) $L_r = t_t/t_s$. Eq. (5) results in the following formula, considering the lifetime-ratio for a *Weibull* distributed failure behaviour

$$R_t(t_t) = R_s(t_s)^{(\chi L_r)^b}. \tag{6}$$

As obtained from eq. (6), the reliability for a given test time depends on the desired product reliability for normal use conditions, the acceleration factor, the lifetime-ratio and the shape parameter.

2. Methods Using Prior Information

The methods described in the following chapters are all based on the *Bayes* theorem (*Cornfield 1967*). In this paper the *Bayes* theorem will be used for determining the amount of test items necessary to ensure the required reliability of a mechanical component.

2.1 Uniform prior

If no prior information is available, the uniform prior pdf can be used within the *Bayes* theorem (*Martz 1982*). Then, the confidence level for the required reliability may be found from

$$C = 1 - \sum_{i=0}^{x} \binom{n + \frac{1}{(\chi L_r)^b}}{i} R_s(t_s)^{(\chi L_r)^b (n-i)+1} (1 - R_s(t_s)^{(\chi L_r)^b})^i \tag{7}$$

where x is the number of failures that occur during the test.

2.2 *Kleyner* et al. Method

Kleyner et al. (*1997*) propose to use a mixture of a uniform distribution and a beta distribution as prior information. The beta distribution results from a former product or preceding tests. The two distributions are combined according to weights of the so-called "knowledge factor". The knowledge factor ρ represents how similar the new and the former product are. For a knowledge factor ρ = 0, there exists no similarity. For a knowledge factor ρ = 1, the information about the former product is totally transferable to the new product. The knowledge factor cannot be determined quantitatively, it has to be estimated.

With regard to the occurrence of failures during the test, the confidence level can be determined by

$$C = \frac{\int_{R_s(t_s)}^{1} \left[\frac{\rho}{\beta(A,B)} R_s(t_s)'^{A-1} (1 - R_s(t_s)')^{B-1} + (1-\rho) \right] R_s(t_s)'^{(\chi L_r)^b (n-x)} \left(1 - R_s(t_s)'^{(\chi L_r)^b}\right)^x \mathrm{d}R_s(t_s)'}{\int_{0}^{1} \left[\frac{\rho}{\beta(A,B)} R_s(t_s)'^{A-1} (1 - R_s(t_s)')^{B-1} + (1-\rho) \right] R_s(t_s)'^{(\chi L_r)^b (n-x)} \left(1 - R_s(t_s)'^{(\chi L_r)^b}\right)^x \mathrm{d}R_s(t_s)'}. \tag{8}$$

Eq. (8) can only be solved numerically.

2.3 *Beyer/Lauster* Method

The *Beyer/Lauster* method (*Beyer 1990*) does not require the entire prior distribution, but considers only one value as prior information - namely the reliability R_0 at the confidence level C_0 = 63.2%. For a test where failures occur, the confidence level can be determined as follows

$$C = 1 - R_s(t_s)^{(\chi L_r)^b n + 1/\ln(1/R_0)} \sum_{i=0}^{x} \binom{n + \frac{1}{(\chi L_r)^b \ln(1/R_0)}}{i} \left(\frac{1 - R_s(t_s)^{(\chi L_r)^b}}{R_s(t_s)^{(\chi L_r)^b}} \right)^i. \tag{9}$$

One has to take into account that the prior information for the *Beyer/Lauster* method results from a test with no failures. Otherwise, it is not valid to use the *Beyer/Lauster* method.

3. Test Procedure for a Mechanical Component

In this practical example the required reliability for a mechanical component has to be ensured. The component passed an accelerated test where the test time was unequal to the required product lifetime. Further tests under real use conditions are to be planned in order to assure the required reliability. The results of the accelerated test are to serve as prior information for the test under real operating conditions. The shape parameter of the *Weibull* distribution is estimated with $b = 1.5$.
The product reliability is required to be 90% at a confidence level of 80%. The first test was an accelerated test where 4 components were tested with an acceleration factor of 3.5 and a lifetime-ratio of 0.5. No failures occurred during the test. Additionally, the uniform distribution is used as prior information for the accelerated test. The test conditions are summarised in Table 1.

Table 1: Test conditions

	acceleration factor χ	lifetime-ratio L_r	sample size n	failures x
accelerated test	3.5	0.5	4	0
test under use conditions	1	1.5	?	?

3.1 Results of the Accelerated Test

Figure 1 shows the confidence level that is obtained from the accelerated test as a function of the cumulative test time. The abscissa is standardised by the test time for one component to conceal the real values of the component lifetime. As can be seen from Figure 1, the confidence level increases during the test period and reaches the value of 66.1% in the end of the accelerated test. Consequently, further tests are necessary to receive a confidence level of 80%.

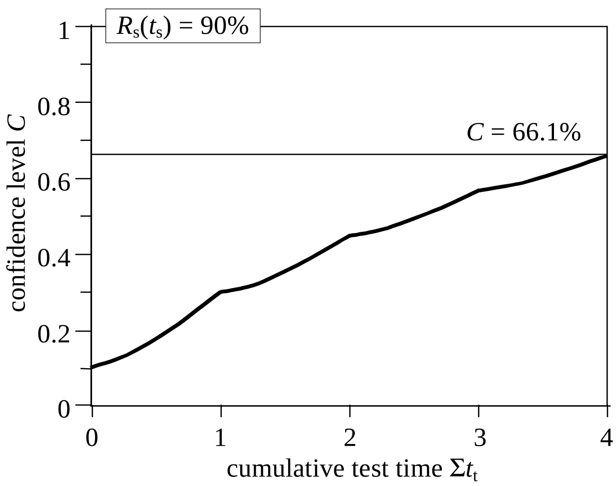

Figure 1: Confidence level C obtained with the accelerated test dependent on the cumulative test time Σt_t (abscissa standardised by the test time for one component)

3.2 Results of the Test Under Use Conditions

The results from the accelerated test are taken into account within the planning of the test under real use conditions by means of *Beyer/Lauster* and *Kleyner* et al.. The sample-size for the test under use conditions was calculated for a test without failures. Depending on the knowledge factor, the sample-size lies between 3 for the best case and 8 for the worst case, Figure 2. The *Beyer/Lauster* method leads to same results as the *Kleyner* et al. method as far as a knowledge factor $\rho = 1$ is concerned.
An unexpected failure occurred during the test. As a result, it was necessary to determine the sample-size again, considering one failure. According to Figure 3 the sample-size becomes 11 for the

Beyer/Lauster method and the *Kleyner* method with a knowledge factor ρ = 1 and 16 for a knowledge factor ρ = 0. Thus, the test under use conditions would have to be performed with at least 11 items to achieve the desired confidence level.

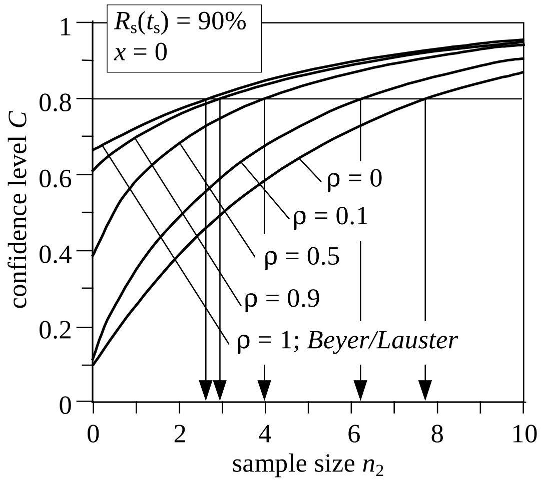

Figure 2: Confidence level as a function of the sample-size for a test without failures

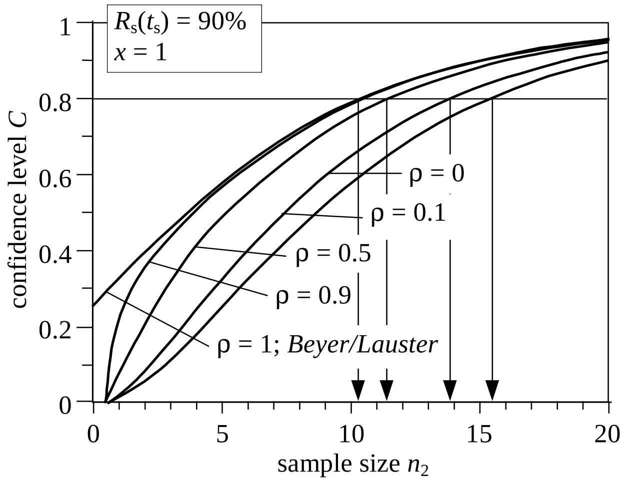

Figure 3: Confidence level as a function of the sample-size for a test with one failure

4. Conclusions

In this paper the acceleration factor was added to the methods by *Kleyner* et al. and *Beyer/Lauster*. Furthermore, the lifetime-ratio was added to the method by *Kleyner* et al.. The methods were compared and discussed within a practical example, where the required reliability for a mechanical component had to be ensured. It was shown that the confidence level increases with higher sample-sizes and knowledge factors. Furthermore, the example showed that the sample-size necessary to demonstrate the product requirements increases for the occurrence of one failure during the test under use conditions.

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