Models to Consider Dependence in Reliability Calculation for Systems Consisting of Mechanical Components

Peter Pozsgai
Institute of Machine Components
University of Stuttgart
D-70569 Stuttgart
Germany
pozsgai@ima.uni-stuttgart.de

Wolfgang Neher
Wolfgang.Neher@gmx.de

Bernd Bertsche
University of Stuttgart
bertsche@ima.uni-stuttgart.de

Abstract

Generally, the methods for the calculation of the system reliability assume independent components with independent failure modes. For mechanical systems in a realistic situation there might be dependencies between the components of the system. Therefore, existing approaches to consider dependencies for the calculation of the system reliability are summarized and extended in our paper. Most of these methods are restricted to small system structures and assume exponential distributions to describe the failure mode, i.e. constant failure rates. It is shown how the models can be used for time-dependent failure rates (e.g. *Weibull* distribution). We describe how to apply the models to bigger system structures.

1. Introduction

Generally, the consideration of the reliability structure and the combination of the components is done by means of the *Boolean* theory. However, its application is limited to systems where the components are – among other assumptions – statistically independent *(Bertsche 1999)*. If the components of the system are not statistically independent, other methods have to be used.

2. Basics & Definitions

2.1 Reliability & Weibull Distribution

The reliability $R(t)$ of a component is the probability that, when operating under stated environmental conditions, the component will perform its intended function adequately for a specified interval of time t. The mathematical description is based on the pdf $f(t)$, the cdf $F(t)$ with $f(t) = \mathrm{d}F(t)/\mathrm{d}t$, the reliability $R(t) = 1 - F(t)$ and the failure rate (hazard function) $\lambda(t) = f(t)/R(t)$. The usual description of the failure behavior of mechanical systems or components is provided by the three-parameter *Weibull* distribution *(Bertsche 1999)*, with the pdf

$$f(t) = \frac{b}{(T-t_0)}\left(\frac{t-t_0}{T-t_0}\right)^{b-1} e^{-\left(\frac{t-t_0}{T-t_0}\right)^b},\ 0 \le t_0 \le t \text{, with} \tag{2.1}$$

shape parameter b, scale parameter T and location parameter t_0. For $t_0 = 0$ one speaks of a two-parameter *Weibull* distribution. The failure rate $\lambda(t)$ of a *Weibull* distribution is a function of time, it can depict all three sections of the well-known bath-tub curve depending only on the shape parameter.

2.2 Dependence Between Components

Dependence between components may occur in systems, if the components are exposed to the same extraordinary stress, if the load is shared by several components in a redundancy structure or if there exists a functional correlation between the system components. These different types of dependence can be classified into common-mode failure, the load-sharing case and the so-called functional dependence *(Neher, 2001)*. Which types of dependence may occur is dependent on the system structure, e.g. in a series system only common-mode failure and functional dependence are possible.

3. Common-Mode Failure

An impact of a single failure mode on an entire redundancy structure is assigned to the class of common-mode failures. Adequate models to describe a common-mode failure are the beta-factor method *(Deutsche Risikostudie Kernkraftwerke 1981)*, the state-graph method *(VDI-Richtlinie 4008, Blatt 9, 1986)* and the fatal shock model by *Marshall & Olkin (1967)*.

3.1 Beta-Factor Method

The model assumes n identical components with failure rate λ_1 in a parallel structure and a redundancy bridging common-mode failure with λ_2. The beta-factor is defined as $\beta = \lambda_2/(\lambda_1 + \lambda_2)$, $0 \leq \beta \leq 1$, ($\beta = 0$: no common-mode failure; $\beta = 1$: only common-mode failure). In other words, the beta-factor represents the conditional probability of being a common-mode failure when a component failure occurs. The reliability of the system yields

$$R_S = \left(1 - \left(1 - e^{-(1-\beta)\lambda t}\right)^n\right) \cdot e^{-\beta\lambda t}, \quad \text{with } \lambda = \lambda_1 + \lambda_2 . \tag{3.1}$$

If the failure rates of the components $\lambda_1(t)$ and the common-mode failure $\lambda_2(t)$ are time-dependent, the beta-factor becomes in general non-constant, $\beta(t) = \lambda_2(t)/(\lambda_1(t) + \lambda_2(t))$. Then the factor can be used to describe the influence of the common-mode failure during the lifetime of the components *(Neher 2001)*. With the *Weibull* distribution the beta-factor is still constant, if both shape parameters have the same value.

3.2 State-Graph Method

In a 1-out-of-2 system with hot standby, with two components A and B, there are three different, independent random failure modes, whereas failure mode S_1 specifies failure of component A, failure mode S_2 specifies failure of component B and S_3 specifies failure of A and B at the same time. Hence, the lifetime of component A and B results in $\tau_A = \min(\tau_1, \tau_3)$ and $\tau_B = \min(\tau_2, \tau_3)$. If the lifetimes are exponential distributed, the state-graph method is equivalent to a time-homogeneous *Markov* process. The system failure probability yields

$$F_S(t) = 1 - e^{-(\lambda_2 + \lambda_3)t} - e^{-(\lambda_1 + \lambda_3)t} + e^{-(\lambda_1 + \lambda_2 + \lambda_3)t}, \tag{3.2}$$

which is equivalent to the beta-factor method with eq. (3.1) if $\lambda_1 = \lambda_2$. The beta-factor method is a special case of the state-graph method. The state-graph method can be applied, if only parts of the system are affected by a common-mode failure or if the system has a more complex structure (not restricted to pure parallel structures) *(Neher 2001)*.

3.3 Fatal Shock Model

The model by *Marshall & Olkin (1967)* describes a system with two components (A and B) and three independent failure modes, where mode 1 destroys A at random time τ_1, mode 2 destroys B at time τ_2 and mode 3 destroys both at τ_{12} (common-mode). The component lifetimes result in $\tau_A = \min(\tau_1, \tau_{12})$ and $\tau_B = \min(\tau_2, \tau_{12})$. The reliability of the system is given by the exponential bivariate distribution

$$R(t_1, t_2) = P[\tau_A > t_1, \tau_B > t_2] = e^{-\lambda_1 t_1 - \lambda_2 t_2 - \lambda_{12} \max(t_1, t_2)} . \tag{3.3}$$

The system is up, if both components are operational. Thus, the model can only be applied to series systems and can be considered as a special case of the state-graph method. *Marshall & Olkin* show how to expand the model for systems up to n components. For a three component series system (common-mode failures λ_{12}, λ_{23}, λ_{13}, and λ_{123}) the reliability is defined by

$$\begin{aligned} R(t_1, t_2, t_3) &= P[\tau_A > t_1, \tau_B > t_2, \tau_C > t_3] \\ &= e^{-\lambda_1 t_1 - \lambda_2 t_2 - \lambda_3 t_3 - \lambda_{12} \max(t_1, t_2) - \lambda_{23} \max(t_3, t_2) - \lambda_{13} \max(t_1, t_3) - \lambda_{123} \max(t_1, t_2, t_3)} . \end{aligned} \tag{3.4}$$

Marshall & Olkin (1967) show how the model can be used, if the failure behavior of the components and of the common-mode failure is described by Weibull distributions, i.e. time-dependent failure rates. Here, the model can also be expanded in general up to n components.

4. Load-Sharing Models

When the load is shared in a redundant system by several components, a failure of one component yields a higher stress of the surviving components. For this case of dependence, several models are known from literature. The *Freund* model *(1961)* will be introduced now in some detail.

4.1 *Freund* Model

The model considers a 1-out-of-2 system with hot standby. The components *A* and *B* have to have constant failure rates α and β. After one component has failed, the surviving component gets a modified (higher) failure rate (α' or β'). The combination of the components' failure density functions to the system density yields a bivariate density function. Integrating yields the cdf of the failure to

$$F_S(t) = \frac{\alpha}{\alpha+\beta}\left(1+\frac{\beta'\cdot e^{-(\alpha-\beta)t}-(\alpha+\beta)\cdot e^{-\beta' t}}{\alpha+\beta-\beta'}\right)+\frac{\beta}{\alpha+\beta}\left(1+\frac{\alpha'\cdot e^{-(\alpha+\beta)t}-(\alpha+\beta)\cdot e^{-\alpha' t}}{\alpha+\beta-\alpha'}\right). \tag{4.1}$$

Because the failure behavior of one component depends on the failure behavior of the other, the marginal distribution of the bivariate distribution can not be exponential. It is possible to expand the *Freund* model for systems with *n* components *(Neher 2001)*. However, the number of possible failure combinations *m* is increasing tremendously ($m = n!$). This means, that the density function consists out of *m* terms and the cdf of the failure can be calculated out of the density function, where each of the *m* terms has to be integrated with a *n*-dimensional integral.

4.2 Other Load-Sharing Models and Comments

Besides the *Freund* model, other known models for the load-sharing case are the state-graph method, the capacity flow model *(VDI-Richtlinie 4008, Blatt 9, 1986)* and the warm standby model *(Koslow & Uschakow 1979)*. All these models allow to model bigger system structures (more than 2 components) but are restricted to constant failure rates. The state-graph method, which is based on a *Markov* process, is the most general one. It is not limited to parallel structures as the *Freund* model, the components may have specific failure rates and the load steps can be chosen individually for every component. The mentioned models for the load-sharing case are not able to consider time-dependent failure rates. In that case, the stochastic process would become non-regenerative and the memory-less property would not be existent any more. An analytic description is possible by partial differential equations, but requires a big effort for derivation and analysis. A more adequate method for the load-sharing case would be the application of simulation techniques.

5. Functional Dependence

The functional dependence considers the influence between the component failure modes, which can be caused by energetic and substantial connections, e.g. contact or same working fluid *(Neher 2001)*.

5.1 Reliability Interval

The approach uses an interval of the system reliability $R_S(t)$ to specify this type of dependence. The bounds of the reliability interval can be determined by considering the extreme cases of total independence and total dependence for series and parallel systems *(Barlow & Proschan 1965)*. For a parallel system, the reliability interval is given by

$$R_S(t) = P\{\max_{1\le i\le n} Y_i > t\} \le 1-\prod_{i=1}^{n}\left(1-P(Y_i > t)\right),\ \text{where } Y_i = i\text{-th component lifetime and} \tag{5.1}$$

Y_S = system lifetime. Here, the *Boolean* model yields the upper bound of $R_S(t)$ with independent components. The case of total dependence reduces the system to one component. Thus, the lower bound of $R_S(t)$ is the component with the highest reliability. A series system yields the component with the lowest reliability as upper bound and the result of the *Boolean* model as the lower bound. For general system structures with functional dependencies the reliability interval is given by

$$\max_{1 \le j \le l} \prod_{Ki \in Tj} R_i(t) \le P\{Y_S > t\} \le \min_{1 \le j \le l} (1 - \prod_{Ki \in Cj} (1 - R_i(t))) \text{, where } T_j = \text{set of minimal paths;} \qquad (5.2)$$

C_j = set of minimal cuts. The upper bound is given by the minimal cut with the lowest reliability and the lower bound is given by the minimal path with the highest reliability. $R_S(t)$ based on the *Boolean* model lies between the two bounds as can be seen from the example in Figure 1 and Figure 2.

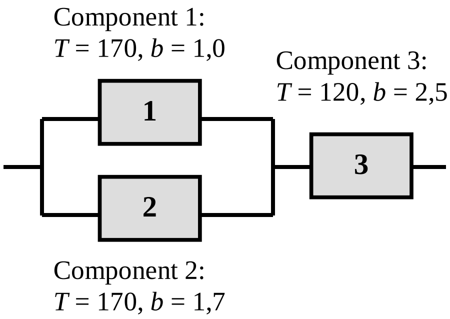

Figure 1: Series-parallel system with three components

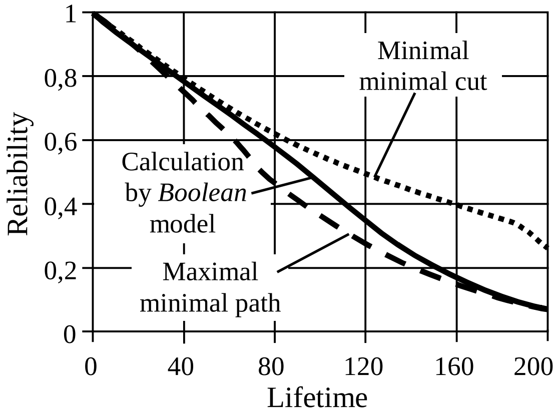

Figure 2: Reliability and its bounds

The method can be applied to components with time-dependent failure rates. It can be used for a system, where the dependence is associated *between* the minimal paths and minimal cuts and also *in* the minimal paths and minimal cuts *(Neher 2001).*

6. Conclusions

The fatal-shock model can be used to consider common-mode failures in a series structure system where the components feature time-dependent failure rates. The beta-factor can be utilized as assessment of risks between independent failure mode and common-mode failure in a parallel structure. If the components have constant failure rates, the state-graph method and the fatal-shock model can be expanded for bigger systems. The presented load-sharing models can only be applied to components with exponential distributions. Here, the state-graph method represents the most general model, which is applicable in an easy way to bigger system structures. The model of functional dependence can be applied to systems with arbitrary structure and components with time-dependent failure rates.

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