

# New Dynamic Reliability Indices for Multi-State System

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## Abstract

In this paper the Multiple-Valued Logic is applied for MSS reliability analysis. MVL approach causes to synthesize special reliability indices. This approach is permitted to investigate influence upon the MSS reliability by the capacity levels of a system component. The new class of Dynamic Reliability Indices had obtained in this paper. We have used the Direct Partial Logic Derivatives of MVL functions for calculation of reliability indices.

## 1. Introduction

Many of modern large-scale systems can perform the task at several different levels. In this case the system failure can lead to decreased capability to perform a given task, but not to complete failure. A system component also can perform its task with some different levels. A system, which can have some different task performance levels, is named *Multi-State System* (MSS). We have shown a MSS of  $n$  components in this paper. The system and its components have  $m$  states (level to performance a task or serviceability).

In this paper we are used structure function approach for reliability analysis. Well know there are many papers in which reliability indices are determined by structure function approach. These indices are used for a system design and its maintenance (*Ushakov 1994*).

As a rule these reliability indices is generalized of indices used for the reliability analysis a systems with two levels to perform a task. Therefore they are not take into account specificity MSS in many cases. So, the indices are obtained for MSS are interest in reliability analysis. One of the approaches for synthesis of these indices is used of the mathematical tools of *Multiple-Valued Logic* (MVL) (*Ushakov 1994*).

The Direct Partial Logic Derivatives of MVL functions for reliability analyses are proposed in *Zaitseva 2001*. The new *Dynamic Reliability Indices* (DRI) for MSS are obtained. The dynamic character of these indices consists on system states for which change of a serviceability level of one of components reduce to the system failure or, in other case, to the repair system. In other words, they define the boundary states of MSS. These indices are determined of MSS structure. It is important at a design stage of a system because allow to receive as reliable so unreliable system states and components influencing to them.

## 2. Dynamic Reliability Indices

DRI include three groups of indices: (a) the Dynamic Deterministic Reliability Indices (DDRI); (b) the Dynamic Integrated Reliability Indices (DIRI); (c) the Component Dynamic Reliability Indices (CDRI).

## 2.1 Dynamic Deterministic Reliability Indices

DDRI are set of the boundary states of MSS. These states are calculated by Direct Partial Logic Derivative for everyone system component. So, a system component is analyzed from a point of view of influence of a modification its serviceability level to a modification of system reliability. Therefore, it is necessary to analyze everyone system components and to check the fact of MSS failure or repairing, if the serviceability level of the component was changed. The Direct Partial Logic Derivative allows to formalize this procedure

**Definition 1.** DDRI are set of MSS states  $\{G\}$  which modify level to performance a task from  $a$  to  $b$  if a component serviceability level (component state) changes from  $j$  to  $k$ . The set of system states  $\{G\}$  is union of subsets  $\{G|_{x_i}\}$ . The subset elements are system states for which a change of a serviceability level of  $i$ -th component from  $j$  to  $k$  causes a change of the system level to performance a task from  $a$  to  $b$ . The mathematical interpretations of DDRI are:

$$\{G\} = \{G|_{x_1}\} \cup \{G|_{x_2}\} \cup \dots \cup \{G|_{x_n}\}, \quad (1)$$

where subsets elements  $\{G|_{x_i}\}$  correspond the values  $t_1 \dots t_{i-1} t_{i+1} \dots t_n$  of variables  $x_1 \dots x_{i-1} x_{i+1} \dots x_n$  for which the values of Direct Partial Logic Derivative  $\partial f(j \rightarrow k) / \partial x_i(a \rightarrow b)$  with respect to  $i$ -th variable are not equal 0:  $\partial f(j \rightarrow k) / \partial x_i(a \rightarrow b) \neq 0$ .

DDRI and algorithm of their calculation are considered in (Zaitseva 2001, Zaitseva 2002) in more detail. Four types of these indices are proposed in these papers. They are calculated with the help of Direct Partial Logic Derivative:  $\partial f(1 \rightarrow 0) / \partial x_i(1 \rightarrow 0)$ ,  $\partial f(1 \rightarrow 0) / \partial x_i(s \rightarrow s-1)$ ,  $\partial f(0 \rightarrow 1) / \partial x_i(0 \rightarrow m-1)$  and  $\partial f(0 \rightarrow 1) / \partial x_i(s-1 \rightarrow s)$  for  $s = 2, \dots, m-1$ .

## 2.2 Dynamic Integrated Reliability Indices

DIRI is a probability evaluation of a modification of level of a serviceability MSS at a changing of a system components serviceability. In particular, the probability of boundary of system states is estimated by these indices. Note, it is necessary to know a structure MSS for an evaluation DIRI only. Therefore, the probability of boundary states of system determine in MSS design. These indices are calculated by DDRI.

**Definition 2.** DIRI are probability of a modification of level to performance a task from  $j$  to  $k$  if a component serviceability level (component state) changes from  $a$  to  $b$

$$g = \sum_{i=1}^n P(i)_{a \rightarrow b}^{j \rightarrow k} \times \prod_{\substack{q=1 \\ q \neq i}}^n (1 - P(i)_{a \rightarrow b}^{j \rightarrow k}), \quad (2)$$

where we have compatible events;  $P(i)_{a \rightarrow b}^{j \rightarrow k}$  is probability of a modification of system level to performance a task from  $j$  to  $k$  if  $i$ -th component serviceability level changes from  $a$  to  $b$ :  $P(i)_{a \rightarrow b}^{j \rightarrow k} = \rho(i)_{a \rightarrow b}^{j \rightarrow k} / m^n$ ;  $\rho(i)_{a \rightarrow b}^{j \rightarrow k}$  is number of system states for which a change  $i$ -th component serviceability level changes from  $a$  to  $b$  causes a modification of system level to performance a task from  $j$  to  $k$ . The  $\rho(i)_{a \rightarrow b}^{j \rightarrow k}$  is number of values of Direct Partial Logic Derivative  $\partial f(j \rightarrow k) / \partial x_i(a \rightarrow b)$  with respect to  $i$ -th variable which are not equal 0. In other words  $\rho(i)_{a \rightarrow b}^{j \rightarrow k}$  is a elements count of set  $\{G|_{x_i}\}$  in (Eq.1).

DIRI are presented in (Zaitseva 2001, Zaitseva 2002) in more detail. They unite four types of indices, which are similar DDRI.

### 2.3 Component Dynamic Reliability Indices

CDRI are a probability evaluation of influence  $i$ -th a system component on a possibility of a failure or repairing system. Most unstable components with a point of view of reliability are determined by these indices. It allows to detect the a unreliable system components for their reservation in MSS design. CDRI are calculated by using DDRI.

**Definition 3.** CDRI are probabilities of MSS failure (repairing) at a modification of a serviceability level of  $i$ -th system component:

$$q(i) = \sum P(i)_{a \rightarrow b}^{1 \rightarrow 0}, \text{ for } \forall a, b \in \{0, 1, \dots, m-1\} \text{ and } a > b, \quad (3)$$

where  $P(i)_{a \rightarrow b}^{1 \rightarrow 0}$  is probability of a compatible events and the system failure if  $i$ -th component serviceability level changes from  $a$  to  $b$ :  $P(i)_{a \rightarrow b}^{1 \rightarrow 0} = P(i)_{a \rightarrow b}^{1 \rightarrow 0} / m^n$ ;  $P(i)_{a \rightarrow b}^{1 \rightarrow 0}$  is number of system states for which a change  $i$ -th component serviceability level changes from  $a$  to  $b$  causes the system failure.

CDRI for MSS failure and repairing coincide. It can be explain by the property of Direct Partial Logic Derivatives (Yanushkevich 1997). This type of indices analyzes connection of MSS components only.

### 3. Calculations of the Dynamic Reliability Indices

Let's consider an example of a Dynamic Reliability Indices calculation of MSS for  $m=3$  and  $n=7$  (see Figure 1a). We used Direct Partial Logic Derivatives of MVL functions for a calculation of the DRI. DRI unite three subclasses of indices: DDRI (see Eq.1), DIRI (see Eq.2) and CDRI (see Eq.3).

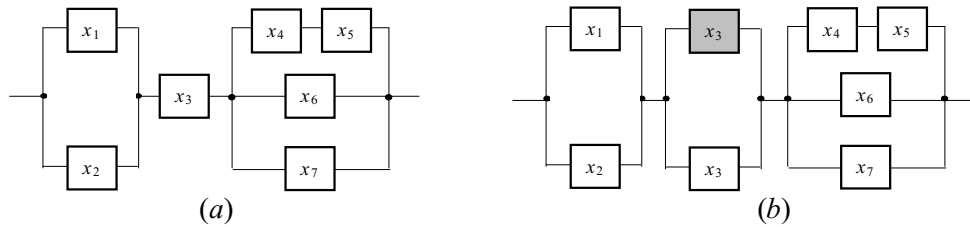


Figure 1: Example of MSS.

The first subclass (DDRI) is considered in (Zaitseva 2001, Zaitseva 2002) in more detail. They are often used in practice. However they have a large calculation complexity. These indices are used as a basis of two next subclasses.

DIRI are probability analogs of DDRI. The applications of these indices are easier than DDRI. For MSS (in a Figure 1a) we have the next DIRI values. The system failure probability at a failure of one of the elements of a system equals  $g_f = 0,3614$ . The probability of system repairing at a replacement of one of the components equals  $g_r = 0,3078$ . The  $\underline{g}_f = 0$  for the elementary (from 2 to 1) drop of components serviceability reduce to system failure. The  $\underline{g}_r = 0$  for the elementary (from 1 to 2) raise of components serviceability reduce to system repairing. They equal zero because MSS has of elementary parallel and sequential junctions only. Therefore the structural function is strongly monotonic in these examples. They do not equal zero for other connections (for example, bridge structures).

CDRI is the last subclass of DRI. CDRI characterizes probability of a system failure (repairing) at a modification of serviceability levels of each of components MSS. The 3-rd component is the most unreliable system component of MSS in Figure 1a. The probability of MSS failure is 0,4526 if the 3-rd component is failure. Therefore for this component is necessary to provide a reserve component in system design (Figure 1b).

In conclusions we shall remark. Well known, the evaluation Direct Partial Logic Derivatives of MVL functions are characterized by high computing complexity. In (*Yanushkevich 1997*) the algorithm of the Test Pattern Generation for Combinational Multi-Valued Networks Based on Direct Partial Logic Derivatives of MVL functions is offered. For use of this algorithm is necessary to present MSS as a combinational multi-valued network. We use two rules for transformation of MSS structure into network: (a) the parallel junction of components of MSS is interpreted as logic element OR; (b) the sequential junction of components of MSS is interpreted as logic element AND. This method for calculation of DRI is represented in (*Zaitseva 2002*).

#### 4. Conclusions

In this paper work we have considered a new class of a MSS reliability indices. They are DRI. They include three subclasses. The first subclass (DDRI) is represented as sets of boundary system states. In other words, if MSS has such state, a modification of a serviceability level of a system component occurs MSS failure or it's repairing. But DDRI have a large calculation complexity. Probability analogies of these indices are often used in practice. These analogies are the second subclass (DIRI). DIRI characterize boundary system states as well as DDRI. The third subclass (CDRI) is presented in this paper for the first time. CDRI allow to estimate influence of each of a system component on a possibility of MSS failure or its repairing. Note, these considered indices allow to estimate a reliability of a MSS structure only. We plan to consider performances and properties of MSS components (for example, cost, time between failures etc.) in more detail in further researches.

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