Modelling the Influence of Maintenance Actions

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Abstract

An operating system is observed to undergo failures. On failure, one of three actions was taken: failures were minimally repaired, given a minor repair or given a major repair. Furthermore, periodically the machine was stopped for either minor maintenance action or major maintenance action. Either on failure or maintenance stoppage, both minor and major repairs are assumed to impact the failure intensity. The issue in this research is to identify the virtual aging process associated with repairs. A series of models appropriate for such an operating/maintenance environment are developed and estimated in order to identify the most appropriate statistical structure. Field data from an industrial setting are used to fit the models.

1 Introduction

In this research, we are concerned with the statistical modelling of repairable systems. Our particular interest is the operation of electrical generating systems. As a repairable system, we assume the failure intensity at a point in time depends on the history of repairs. In the environment under investigation, it was observed that maintenance decisions were regularly carried out. We assume that such actions impacted the failure intensity. Specifically we assume that maintenance actions served to adjust the virtual age of the system in a Kijima (1989) Typ manner. Kijima proposed that the state of the machine just after repair can be described by its so-called virtual age which is smaller (younger) than the real age. In his framework, the failure rate depends on the virtual age of the system. Furthermore a repair, following a failure, can be a minimal repair or a maintenance action.

Kijima proposed two repair effect models. In his first model he assumed that repairs served only to remove damage created in the last sojourn (a Kijima Typ 1 virtual age process). In his second model he assumed that the repair action could remove all damage accumulated up to that point in time (a Kijima Typ 2 virtual age process). That is, such repairs reset the virtual age of the unit to somewhere between that of a completely restored unit (good-as-new repair) and a minimally repaired unit. The concept of minimal repair upon failure is well understood in the literature (Brown 1972) and the resultant non-homogeneous Poisson process has been applied extensively to describe the operation of repairable systems. Incomplite repair processes provide a more general framework for the modelling of failure-repair process (Love & Guo 1994).

Consistent with the data being used to test this model, two levels of maintenance action were observed. Periodically the unit was stopped for minor repairs. Less frequently the unit was stopped for more extensive, major repairs. Unlike the more typical treatments we do not assume in this research that major repairs served to reset the failure intensity of the system (a good-as-new repair). Such an assumption appears to be an excessive constraint on the calibration of such systems. Hence we treat the impact of major repairs as also producing a reduction in virtual age lying between renewal and minimal. We will further assume that major repairs have more impact than minor repairs. Despite regular shutdowns for maintenance however, the system was observed to periodically fail. Repair activity for such failures took one of three forms. With some failures, no repair work was reported and as such they are assumed to have been minimally repaired. Following other failures however repair work was undertaken and was reported as either a minor or a major repair. The purpose of this paper is to provide a model to estimate the impacts of these various activities.

2 Modelling the System

Consider the impact of repairs. A system (machine) starts working with an initial prescribed failure rate $\lambda_1(t) = \lambda(t)$. Let t_1 denote the random time of the first sojourn. At this time t_1 the item will be repaired with the degree $\xi_{11}\xi_{21}$. The two degrees are used to distinguish between three possible types of repair: When the system is minimally repaired then both degrees are equal to one. When we had a minor repair then we set $\xi_{21} = 1$, $\xi_{11} = \xi_1$, where ξ_1 is the (unknown) impact of a minor repair. When the system was major repaired then we have the composite impact $\xi_{21} = \xi_2$, $\xi_{11} = \xi_1$. Notice here we assume that major repairs are modelled as a composite effect $(\xi_1\xi_2)$ not simply ξ_2.
The virtual age of the system at the time t_1, following the repair, is $v_1 = \xi_{11}\xi_{21}(t_1)$, implying the age of the system is reduced by maintenance actions. The distribution of the time until the next sojourn then has failure intensity $\lambda_2(t) = \lambda(t - t_1 + v_1)$. Assume now that t_k is the time of the k^{th} $(k \geq 1)$ sojourn and that $\xi_{1k}\xi_{2k}$ is the degree of repair at that time. We assume that $0 \leq \xi_{1k} \leq 1$, $0 \leq \xi_{2k} \leq 1$, $\xi_{1k} \in \{1, \xi_1\}$, $\xi_{2k} \in \{1, \xi_2\}$ and $\xi_{1k}\xi_{kn} \in \{1, \xi_1, \xi_1\xi_2\}$ for $k \geq 1$.
After repair the failure intensity during the $(k+1)^{th}$ sojourn is determined by

$$\lambda_{k+1}(t) = \lambda(t - t_k + v_k) \quad , t_k \leq t < t_{k+1}, k \geq 0,$$

where $v_k = \xi_{1k}\xi_{2k}(v_{k-1} + (t_k - t_{k-1}))$.
The process defined by $v(t, \xi_1, \xi_2) = t - t_k + v_k$, $t_k \leq t < t_{k+1}$, $k \geq 0$ is called the *virtual age process* (Last and Szekli 1995).

For estimation purpose it is necessary to differentiate between sojourn numbers associated with failure times and those associated with maintenance (censor) times. Let δ_k be an indicator with

$$\delta_k = \begin{cases} 1 & \text{if the sojourn number } k \text{ is related to failure time,} \\ 0 & \text{if the sojourn number } k \text{ is related to censored observation.} \end{cases}$$

In this paper we assume that the baseline failure intensity of the system follows a Weibull distribution

$$\lambda(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1}, \; \beta > 0, \; \alpha > 0 \; .$$

Our purpose is to estimate 4 parameters; α, β, ξ_1 and ξ_2.

3 Parameter Estimation

3.1 The Log-likelihood Function

The loglikelihood function for observation of point processes is of the form (Liptser & Shiryayev 1978)

$$\ln L(t;\theta) = \sum_{k=1}^{N(t)} \ln\Big(\lambda(t_k - t_{k-1} + v_{k-1})\Big)^{\delta_k} + \int_{t_0}^{t} (1 - \lambda(v(x)))dx.$$

The first term contains all failures and the second term contains the information about working periods without failures. To simplify the notation let us denote by $t_{N(t)+1} = t$ the end of the observations. Define $\mathcal{I}$ as $\int_{t_0}^{t} \lambda(v(x))dx$. To calculate $\mathcal{I}$ we can use the fact that $v(x)$ is linear between two consecutive failure times. Thus we get the following LL function:

$$\begin{aligned}\ln L(t;\theta) \quad &= \quad \tilde{N}(t)(\ln\beta - \beta\ln\alpha) + (\beta-1)\sum_{k=1}^{N(t)} \delta_k \cdot \ln(t_k - t_{k-1} + v_{k-1}) \\ &\quad - \frac{1}{\alpha^\beta}\sum_{k=1}^{N(t)+1} \Big((t_k - t_{k-1} + v_{k-1})^\beta - (v_{k-1})^\beta\Big) + t, \qquad (1)\end{aligned}$$

where $\tilde{N}(t)$ denotes the number of failures until t, $\tilde{N}(t) = \sum_{k=1}^{N(t)} \delta_k$. Using the standard maximum likelihood approach to maximize equation (1) some analytical simplifications of the structure are possible. First we can see that it is possible to explicitly determine parameter α:

$$\hat{\alpha} = \left(\frac{S_2}{\tilde{N}(t)}\right)^{1/\beta}, \quad S_2 = \sum_{k=1}^{N(t)+1} \Big\{(t_k - t_{k-1} + v_{k-1})^\beta - (v_{k-1})^\beta\Big\}$$

and the remaining log likelihood function is

$$\ln L(t,\theta) \sim \tilde{N}(t)\ln\beta - \tilde{N}(t)\ln S_2 + (\beta-1)\sum_{k=1}^{N(t)} \delta_k \ln(t_k - t_{k-1} + v_{k-1}) \,,$$

wich depends on the 3 unknown parameters β, ξ_1, and ξ_2. The parameters ξ_1 and ξ_2 implicitly define the virtual ages $v_1, v_2, \ldots, v_{N(t)}$. Now for any set of observations it is possible to maximize the loglokelihood function numerically (specifically in our case, the use of the Gauss package).

Maximum Likelihood estimation also permits the determination of confidence intervals for the unknown parameters (or simultaneous confidence regions for two or more parameters). The most efficient way to calculate confidence estimates is based on the likelihood ratio. It is well known (see e.g. Barndorff-Nielsen and Blæsild, 1986)), that in general the loglikelihood ratio

$$w = 2\{\log L(\hat{\theta}) - \log L(\theta)\}$$

is asymptotically χ^2-distributed with k degrees of freedom, where k is the number of parameters of interest, $\hat{\theta}$ is the estimate of all parameters and θ is the estimate of the nuisance parameters if the parameters of interest are fixed. This fact can be used to calculate confidence intervals or regions (and is directly obtainable from Gauss).

4 Analysis of Hydro-electric Turbine Data

Our immediate interest in the development of these estimation procedures was to obtain an operating/repair effects model consistent with data obtained from hydro-electric turbines. Operating data from one specific turbine of the B.C. Hydro Power system was used to test these procedures. This set contained information over the period January 1977 through December 1997. In all, 496 sojourns were recorded with 160 failures. Maximum likelihood estimation and (asymptotic) confidence intervalls of the parameters (α, β, ξ_1, and ξ_2) were evaluated with the following results:

Table 1: Point and confidence estimates

Par.	estimate	1% low. b.	5% low. b.	5% upp. b.	1% upp. b.
α	115 003.08	96 746.59	100 697.33	132 117.27	138258.78
β	1.212	1.1095	1.1345	1.2875	1.3105
ξ_1	1.0002	.6866	.7989		
ξ_2	.5457	.1096	.1917	.7733	.8239

In the presentation the results of an simulation study and some other models for modelling the effect of repair actions are presented.

We conclude by noting that the environment studied here involved a complex set of operating/maintenance activities. Identifying and calibrating an appropriate model has important ramifications for subsequent maintenance planning.

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