Estimating Distribution Functions Subject to a Stochastic Precedence Constraint

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Abstract

For any two random variables X and Y with distributions F and G de¯ned on [0; 1), X is said to stochastically precede Y if P(X · Y) ¸ 1=2. For independent X and Y, stochastic precedence is equivalent to E[G(X ¡)]leq1=2. The applicability of stochastic precedence in a variety of statistical contexts, including reliability modeling, tests for distributional equality vs. various alternatives and the relative performance of comparable tolerance bounds, is discussed. Two approaches for estimating the underlying distributions of under the assumption that they obey a stochastic precedence constraint. One is based on data shrinkage and the other involves data translation, and each is shown to lead to a root n-consistent estimator of the underlying distribution. The asymptotic behavior of each of the estimators is fully characterized. Conditions are given under which each estimator is asymptotically equivalent to the corresponding empirical distribution function, or, in the case of right censoring, the Kaplan Meier estimator. In the complementary cases, evidence is presented, both analytically and via simulation, which demonstrates that the new estimators tend to outperform the edf when sample sizes are su±ciently large. Finally, a two sample procedure is proposed for testing the presence of stochastic precedence between two distributions. The test is a modi¯ed version of the Wilcoxon-Mann-Whitney rank test, and Monte Carlo results show the test for stochastic precedence is especially useful in applications in which stochastic precedence holds but stronger stochastic orders do not. Asymptotic unbiasedness and consistency of the proposed test are established.