Models of Software Reliability Based on Time Lags between Failures and Fault Corrections

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Abstract

For the most part of software reliability models it is assumed that when a failure arises, the fault which caused it is immediately removed. However this assumption is not always realistic. The aim of this work is to propose a model which is different of that introduced in (Soler 1996) and which is based on time-lagged point processes.

1 Introduction and Notations

In many recent works the point processes are the preferable tool for modeling the evolution in time of the software (Kuo and Yang 1996), (Chen and Singpurwalla 1997), (Al-Mutairi, Chen, and Singpurwalla 1998). One of the qualities of such an approach is that using point processes some classical software reliability models are unified.

An important assumption made in almost all software reliability models is: when a failure arises the corresponding fault is immediately removed. However, this assumption is not always verified; in several cases the correction time is observable, while the corresponding failure time is not. In these aforementioned cases, the correction takes place in a random period of time after the failure. Motivated by that Soler introduced in (Soler 1996) a software reliability model, in which failure times and correction times generate two different point processes.

Let's consider the following notations:

- $0 \leq T_1 \leq T_2 \leq ...$ are the failure times, $X_i = T_i - T_{i-1}$ are the interfailure times and $N_t = \sum_{i=1}^{\infty} I_{[0,t]}(T_i)$ is the counting process associated to the failures i.e. the failure process.
- $0 < C_1 \leq C_2 \leq ...$ are the correction times, $Y_i = C_i - C_{i-1}$ are the times between two corrections and $K_t = \sum_{i=1}^{\infty} I_{[0,t]}(C_i)$ is the counting process associated to the corrections i.e. the correction process.

2 Models of software reliability based on non-coinciding correction and failure processes

In this section we briefly describe the model proposed in (Soler 1996). We also introduce a new model of software reliability based on the idea of different failure and correction processes.

2.1 The Soler model

Assumptions:

1. The failures are not blocking;
2. A correction takes place after "too many" failures were experienced;
3. Between two successive corrections the failure rate is constant and the failure times form an homogeneous Poisson process;

4. The processes $\{N_t\}_{t\geq 0}$ and $\{K_t\}_{t\geq 0}$ are mutually exciting.

The fourth assumption is concretized in the form of the intensity μ_t of the correction process depending on the history of both processes:

$$\mu_t = \mu(N_t - N_{C_t})$$

With these hypotheses $\hat{\mu}_t$ is determined, where $\hat{\mu}_t$ is the intensity of the correction process depending only of its history. It is shown that the times Y_i between two successive corrections, are independent variables with density functions:

$$f_{Y_i}(y) = \lambda_i(1 - e^{-\mu y})e^{-\lambda_i\left(y - \frac{1-e^{-\mu y}}{\mu}\right)} \quad i = 1, 2, ...$$

In (Soler 1996) the form which the testing program has between the correction times C_{i-1} and C_i is named the ith *version* of the program. Hence Y_i is in fact the lifetime of the ith version.

The parameters λ_i and μ are estimated considering the lifetimes of the versions and possibly the number of failures in each version to be known.

2.2 The new proposed model

In order to describe the characteristics of the new model we need first to specify the notion of point process with order statistic (OS) property (Crump 1975), (Berg and Spizzichino 1999).

Let's consider $\{N_t\}_{t\geq 0}$ a point process with arrival times $T_1 \leq T_2 \leq ...$ and with the following properties: $N_0 = 0$; $E\{N_t\} = M_t < \infty \quad t \geq 0$; $M'_t > 0$ and bounded on finite intervals for $t > 0$. Then N_t has *the order statistic property* if, given that $N_t = n$, $T_1 \leq T_2 \leq ... \leq T_n$ are distributed like the order statistics of n i.i.d. variables with distribution function $F_t(x)$.

In (Crump 1975), among other characteristics of the point processes with OS property, it is shown that these processes are in fact nonhomogeneous birth processes.

Now, for our model, we suppose that the failure process has the OS property. Such an assumption is motivated by the fact that for many software reliability models, the failure times can be viewed as order statistics (Kuo and Yang 1996), (Raftery 1987). Moreover we assume that the correction process is triggered by the failure process. More exactly, if a failure is experienced at the moment T_i, then the correction which removes the fault associated to this failure will take place after a random time period W_i. If $U_i = T_i + W_i$, $i = 1, 2, ...$, then the correction times $C_i = U_{(i)}$, $i = 1, 2, ...$ The failure process is an unobservable triggering process (TP) and the correction process is an observable impact process (IP). In (Berg and Spizzichino 1999) it is proved that if the TP has the OS property, then IP has the OS property.

We consider known: the distribution function F_t, the probability $P_t(n) = P(N_t = n)$ and the distribution function G of W_i, $i = 1, 2, ...$ Moreover we assume that $i - 1$ failures and $j < i - 1$ corrections were experienced. Then, our aim is to compute the probability of no failure in a certain interval of time after the jth correction: $P(T_i - C_j > t|C_j = c_j)$ and to make predictions about the future of the failure process computing $P(N_{t_1} = n_1|N_t = n, K_t = k)$.

Even if the computations are rather complicated and therefore this model is more theoretical than practical, we think that it has its importance. That is because it is the second model proposed for software reliability in which the failure process is different from the correction process. Also analyzing particular cases of processes with OS property, this model could be a starting point for other more practical models of the same type.

Acknowledgments

I would like to thank Professor Fabio Spizzichino for the fruitful talks which we had and for the very kind encouragements.

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